

# **In the midst of Chaos, good predictions: How Meteorology manages Uncertainty**

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**Eugenia Kalnay**

S-C. Yang, M. Peña, M. Hoffman, E. Lynch, S. Sharma, K. Ide, T.  
Miyoshi, S. Greybush, S. Penny, ...

and the Weather-Chaos Group

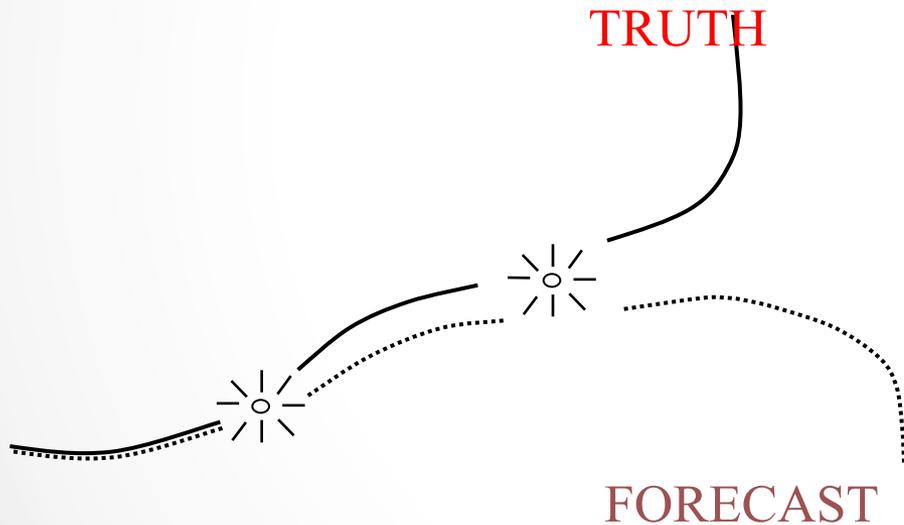
**University of Maryland**

Central theorem of chaos (Lorenz, 1960's):

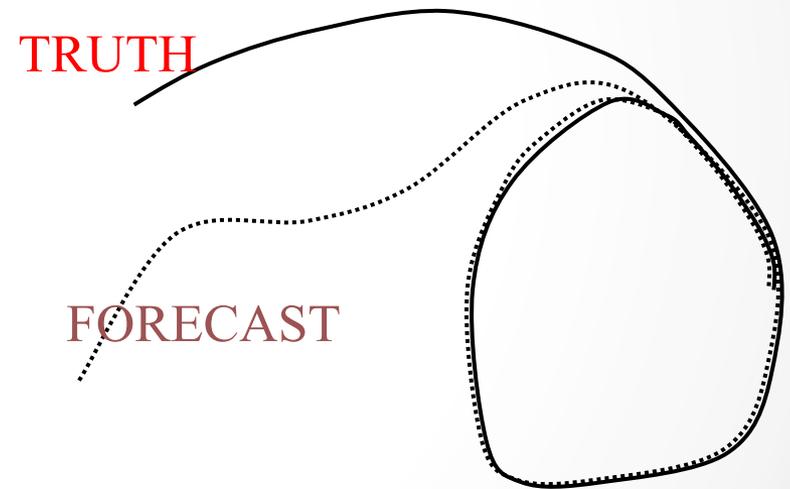
a) **Unstable** systems have **finite predictability** (chaos)

b) **Stable** systems are **infinitely predictable**

a) Unstable dynamical system



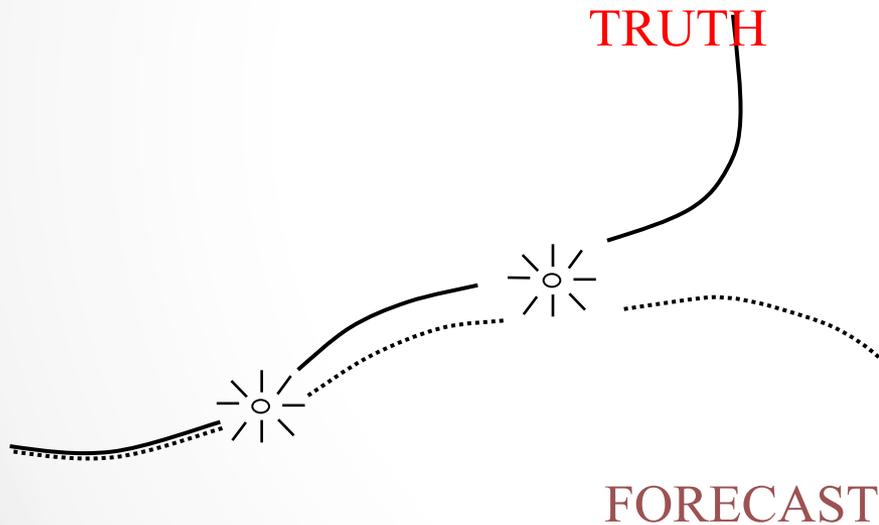
b) Stable dynamical system



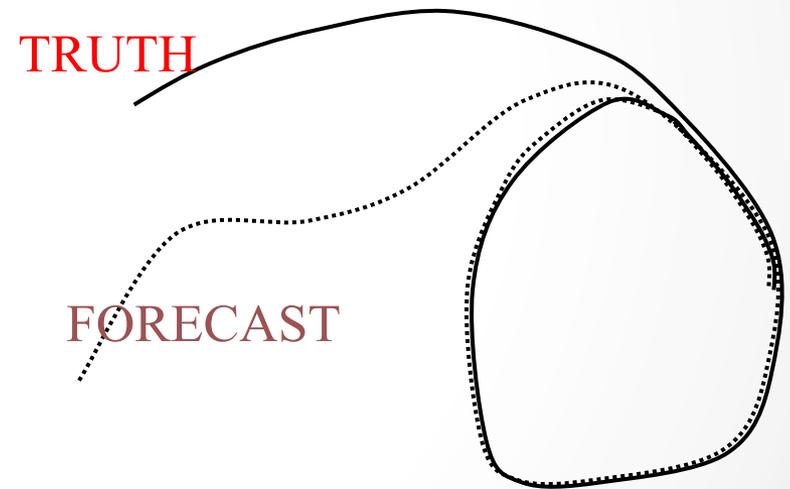
Central theorem of chaos (Lorenz, 1960's):

- a) **Unstable** systems have **finite predictability (chaos)**
- b) **Stable** systems are **infinitely predictable**

a) Unstable dynamical system



b) Stable dynamical system



We developed a simple method to find the **instabilities** (“breeding”)

# **Definition of Chaos**

**(Lorenz, March 2006, 89 years old)**

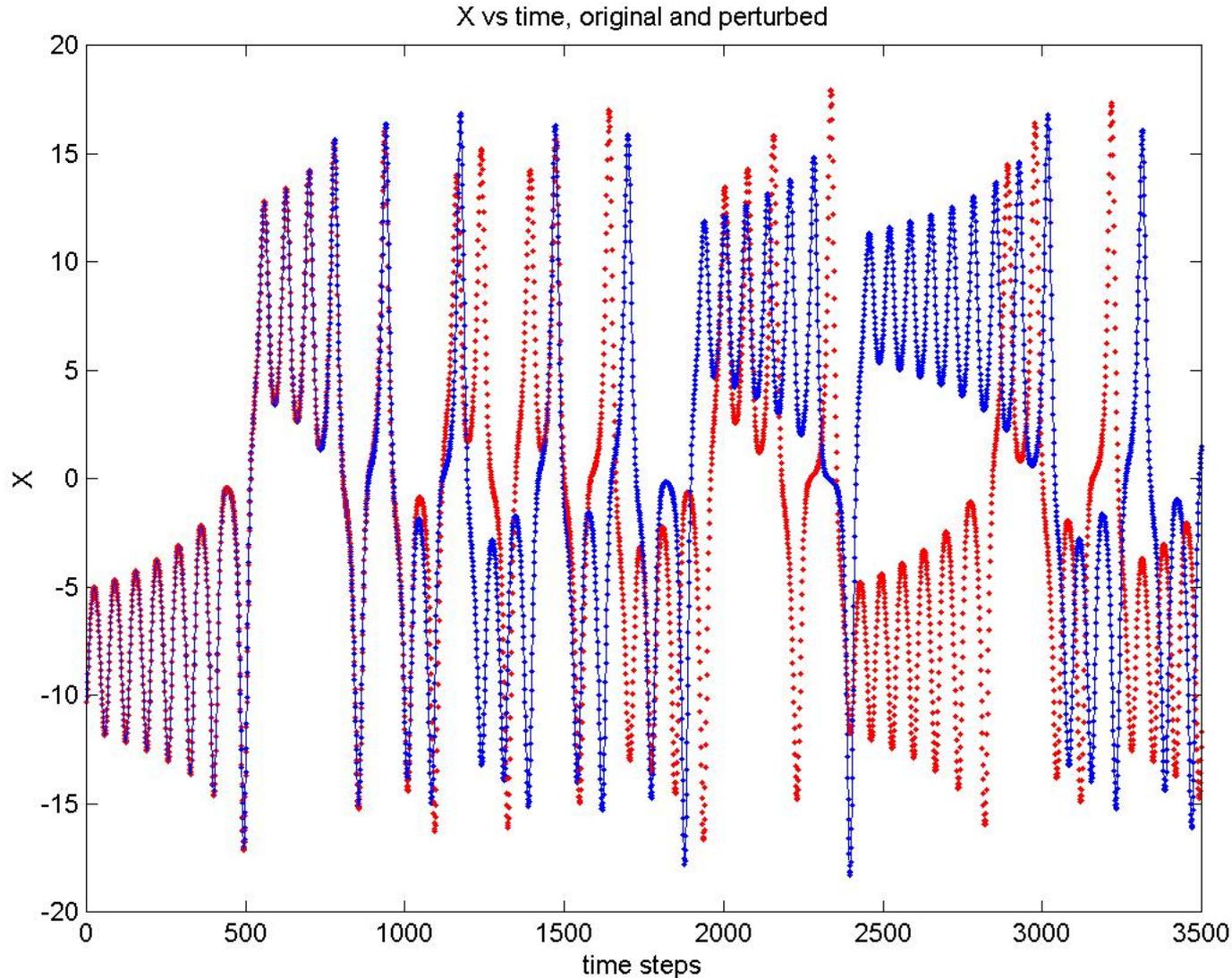
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**WHEN THE PRESENT DETERMINES  
THE FUTURE**

**BUT**

**THE APPROXIMATE PRESENT DOES NOT  
APPROXIMATELY DETERMINE THE FUTURE**

# The approximate present does not approximately determine the future!

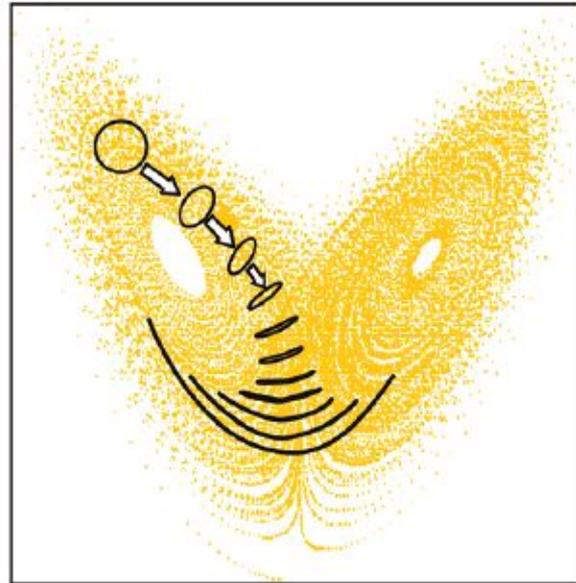


Predictability depends on the initial conditions (Palmer, 2002):

stable



less stable



unstable



**Errors with unstable initial conditions  
(with “growing errors of the day”)  
grow much faster**

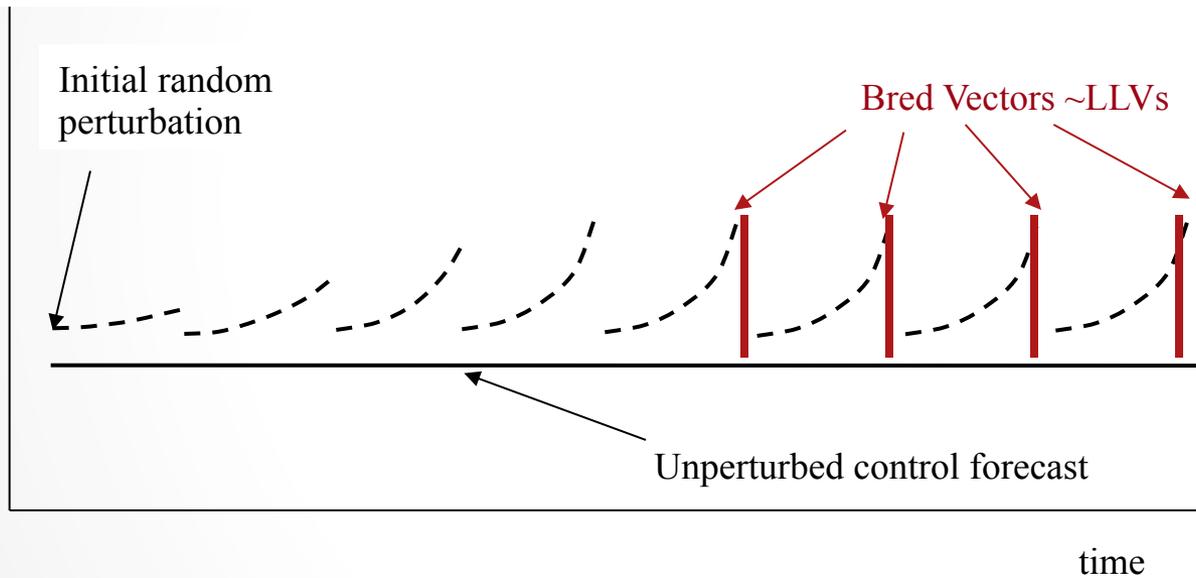
# An 8 week RISE project for undergraduate women (2002)

- We gave a team of 4 RISE intern undergraduates a problem: Play with the famous Lorenz (1963) model, and explore its predictability using “breeding” (Toth and Kalnay 1993), a very simple method to study the growth of errors.
- We told them: “Imagine that you are forecasters that live in the Lorenz ‘attractor’. Everybody living in the attractor knows that there are two weather regimes, the ‘**Warm**’ and ‘**Cold**’ regimes. But what the public needs to know is **when** will the change of regimes take place, and **how long** are they going to last!!”.
- “Can you find a forecasting rule to alert the public that there is an **imminent** change of regime?”

# Breeding: simply running the nonlinear model a second time, from perturbed initial conditions.

Only two tuning parameters: rescaling amplitude and rescaling interval

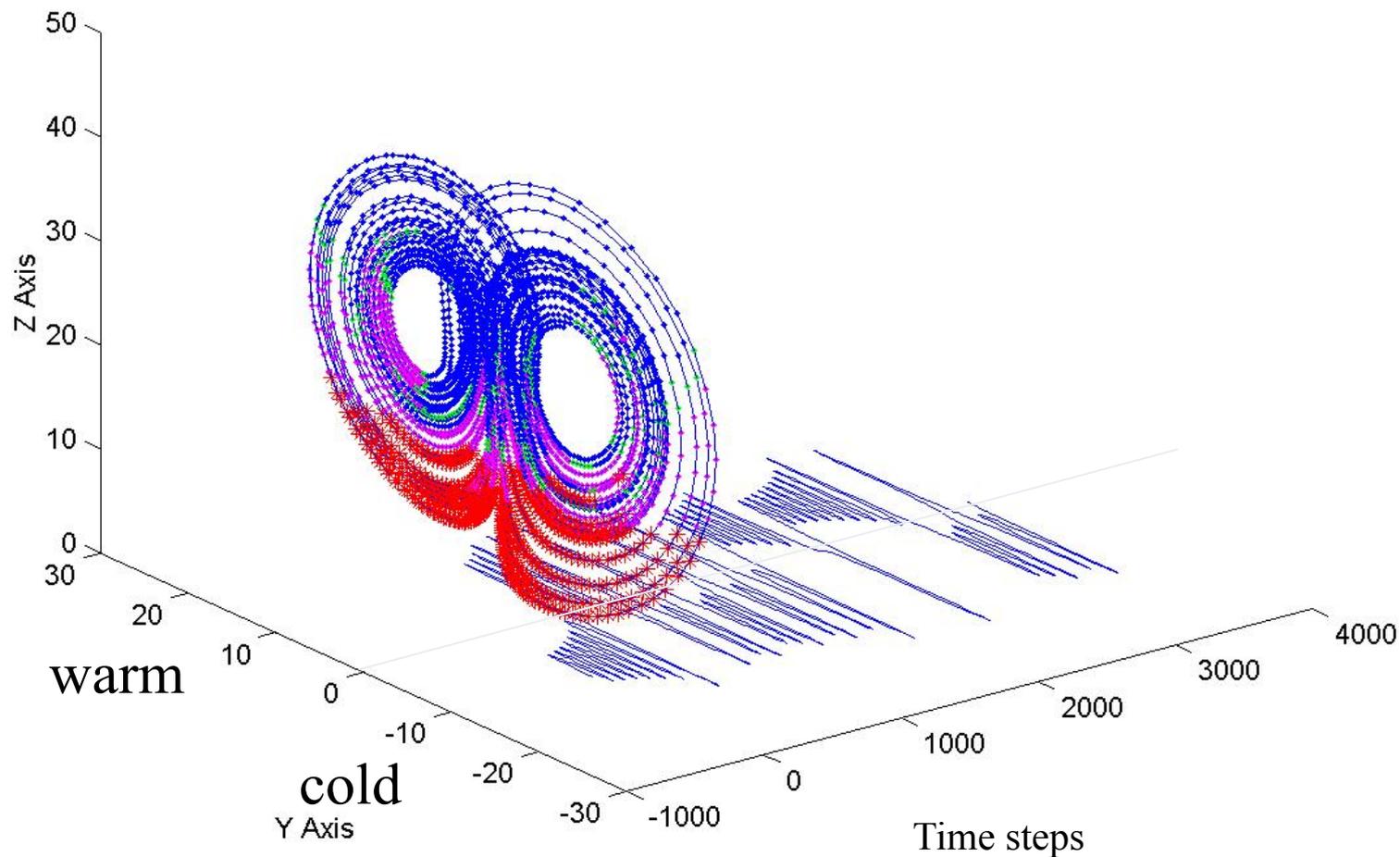
Forecast values



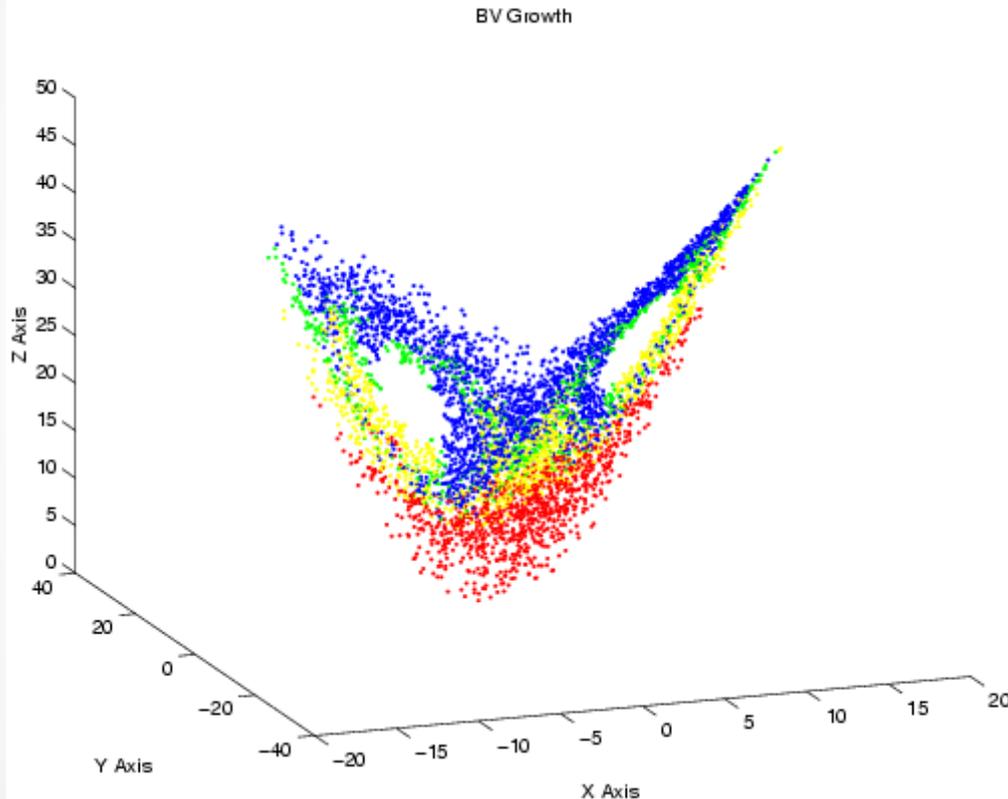
Local breeding growth rate:

$$g(t) = \frac{1}{n\Delta t} \ln \left( \frac{|\delta \mathbf{x}|}{|\delta \mathbf{x}_0|} \right)$$

**4 summer interns computed the Lorenz Bred Vector  
growth rate: red means large BV growth,  
blue means perturbations decay**



In the 3-variable Lorenz (1963) model we used breeding to estimate the local growth of perturbations:

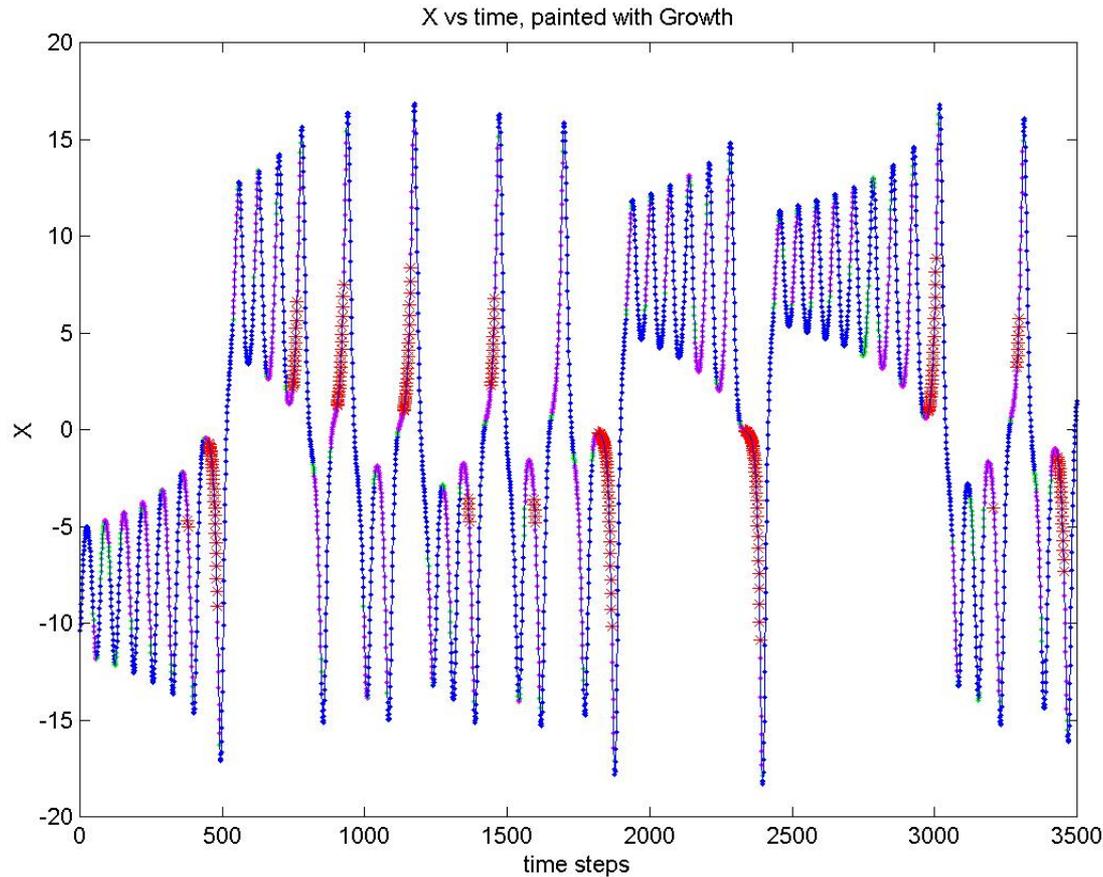


Bred Vector Growth:  
red, high growth;  
yellow, medium;  
green, low growth;  
blue, decay

With just a single breeding cycle, we can estimate the stability of the whole attractor (Evans et al, 2004).

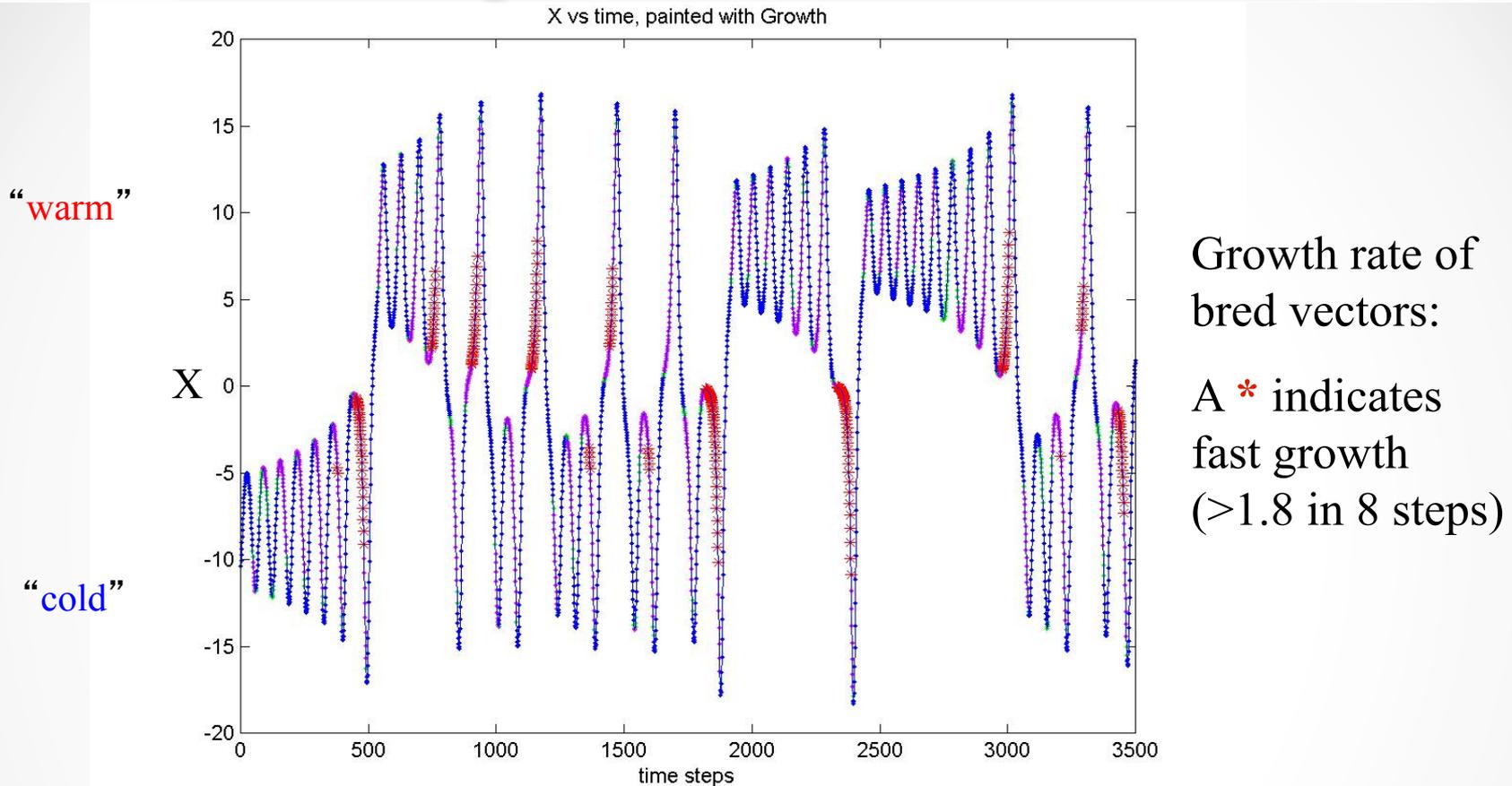
**This looked promising, so we asked the interns to “paint”  $x(t)$  with the bred vector growth, and the result almost made me faint:**

**This looked promising, so we asked the interns to “paint”  $x(t)$  with the bred vector growth, and the result almost made me faint:**



Growth rate of bred vectors:  
A \* indicates fast growth (>1.8 in 8 steps)

# Forecasting rules for the Lorenz model:



Regime change: The presence of **red stars** (fast BV growth) indicates that the next orbit will be the **last one in the present regime**.

Regime duration: **One or two red stars**, next regime will be short. **Several red stars**: the next regime will be long lasting.

**These rules surprised Lorenz himself!**

**Can we apply these ideas to a physical system for which we don't know the model? Yes we can!**

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**Breeding Vectors in the Phase Space Reconstructed from Time Series Data**

**Erin Lynch, D. Kaufman, S. Sharma, E. Kalnay and K. Ide (2014)**

# Summary so far and rest of the talk

- Breeding is a simple generalization of Lyapunov vectors, for finite time, finite amplitude: simply run the model twice, take the difference and rescale...
- Breeding in the Lorenz (1963) model gives accurate forecasting rules for the “chaotic” change of regime and duration of the next regime that surprised Lorenz himself!
- Can be applied to **real** time series **without knowing the model**

## Rest of the talk:

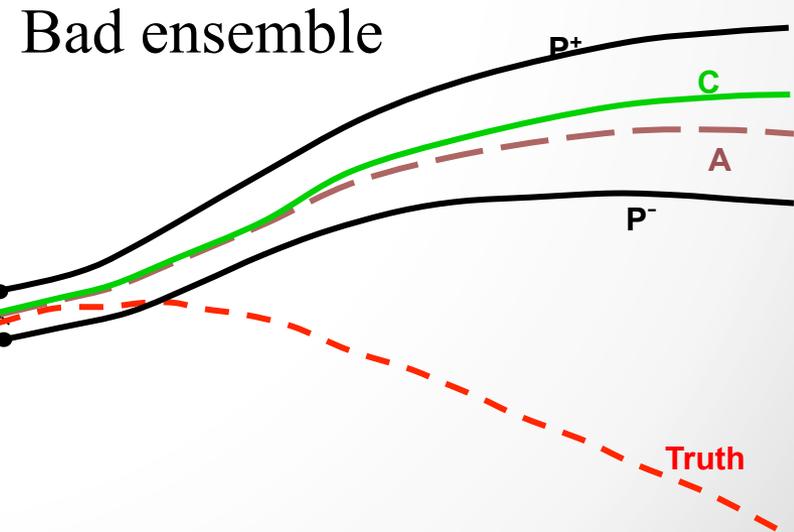
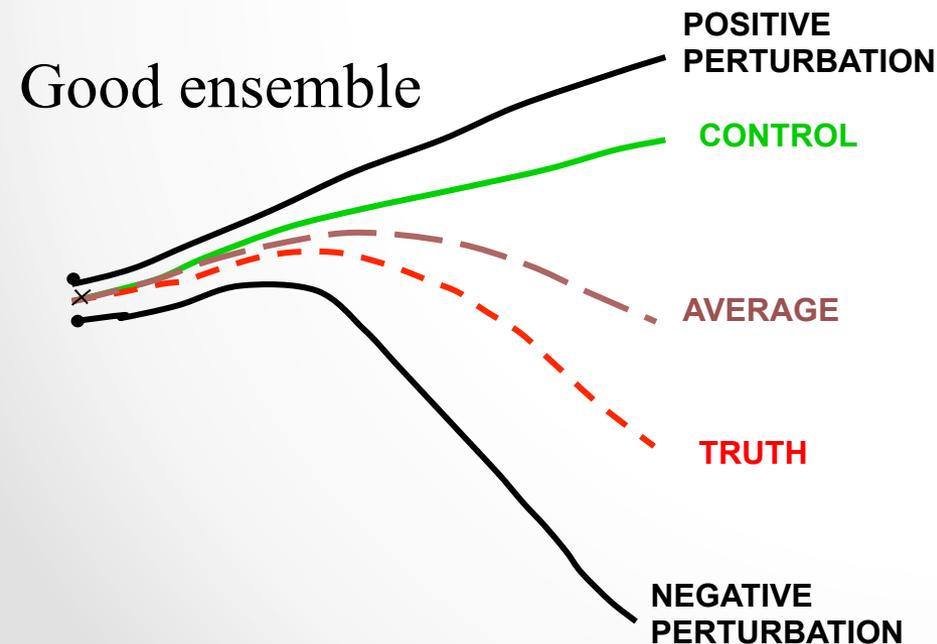
- **The same ideas can be applied to fight chaos in the full forecast models that have dimension 100 million rather than just 3!**
- In the atmosphere, in the ocean, in Mars, and in coupled systems
- We can also use breeding to understand the **physical mechanisms** of the instabilities that create chaos.

# A major tool to “fight chaos” is ensemble forecasting

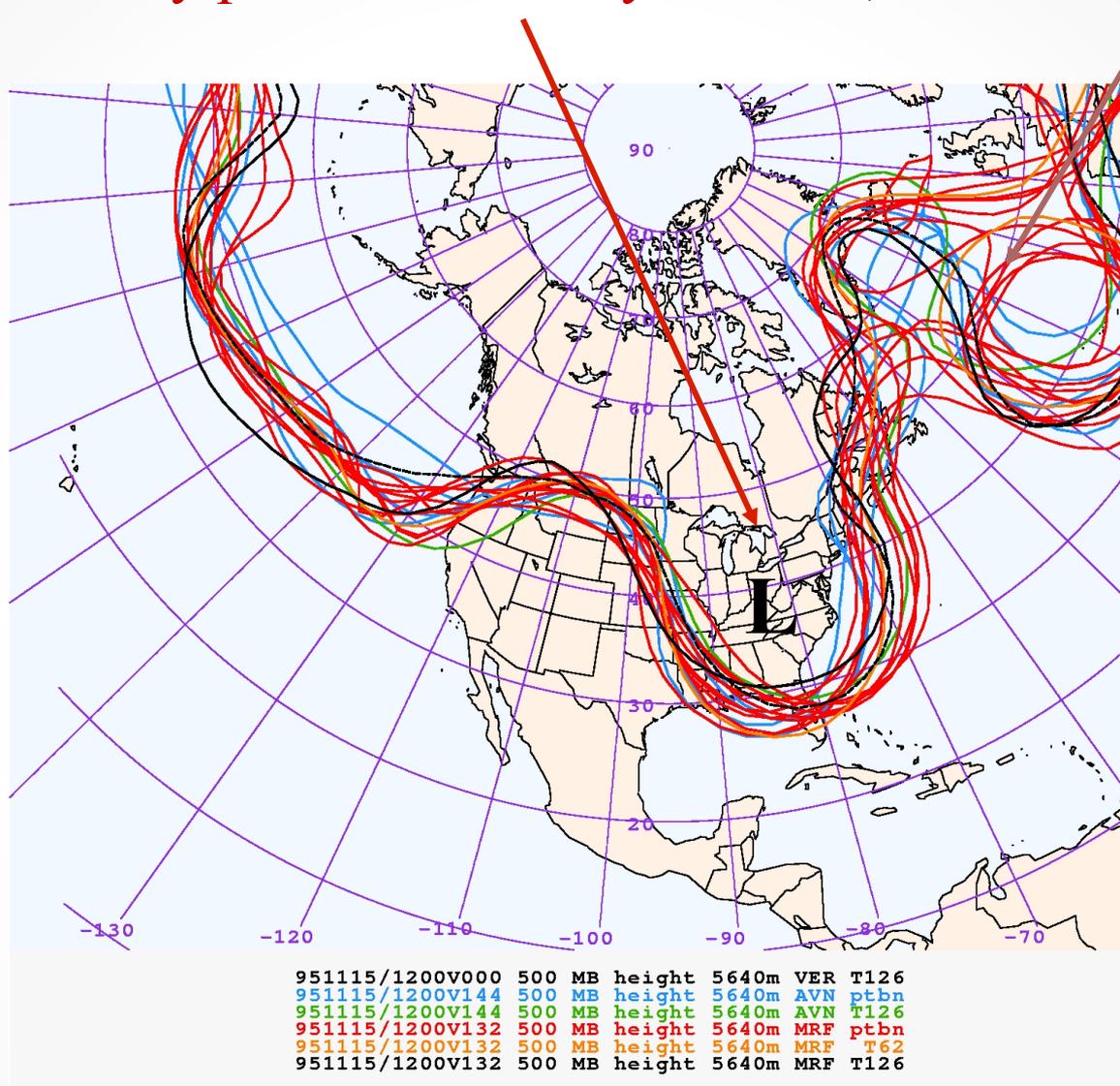
An ensemble forecast starts from initial perturbations to the analysis...

In a good ensemble “truth” looks like a member of the ensemble

The initial perturbations should reflect the analysis “errors of the day”.



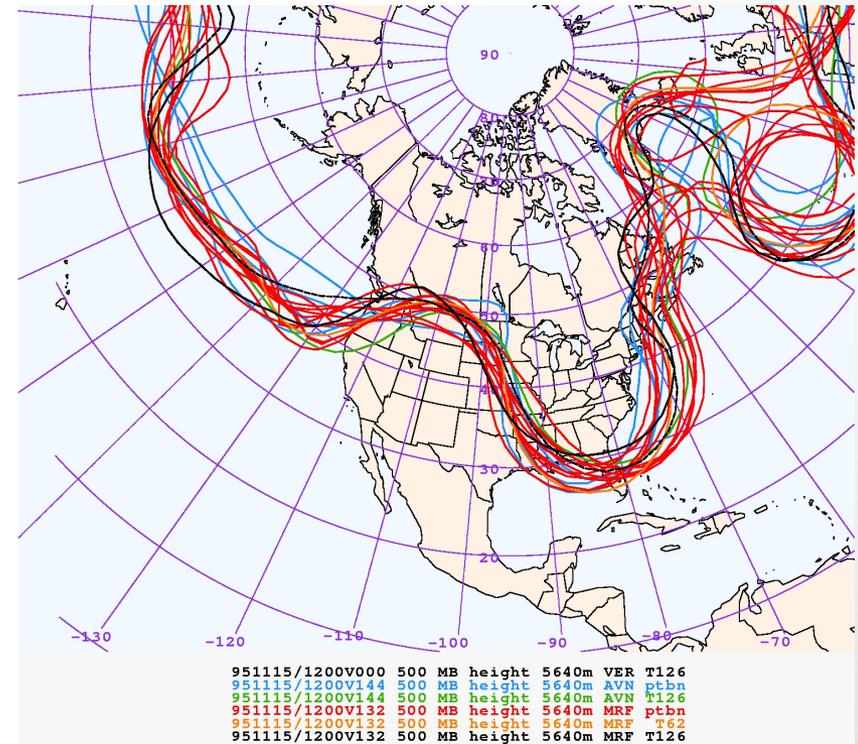
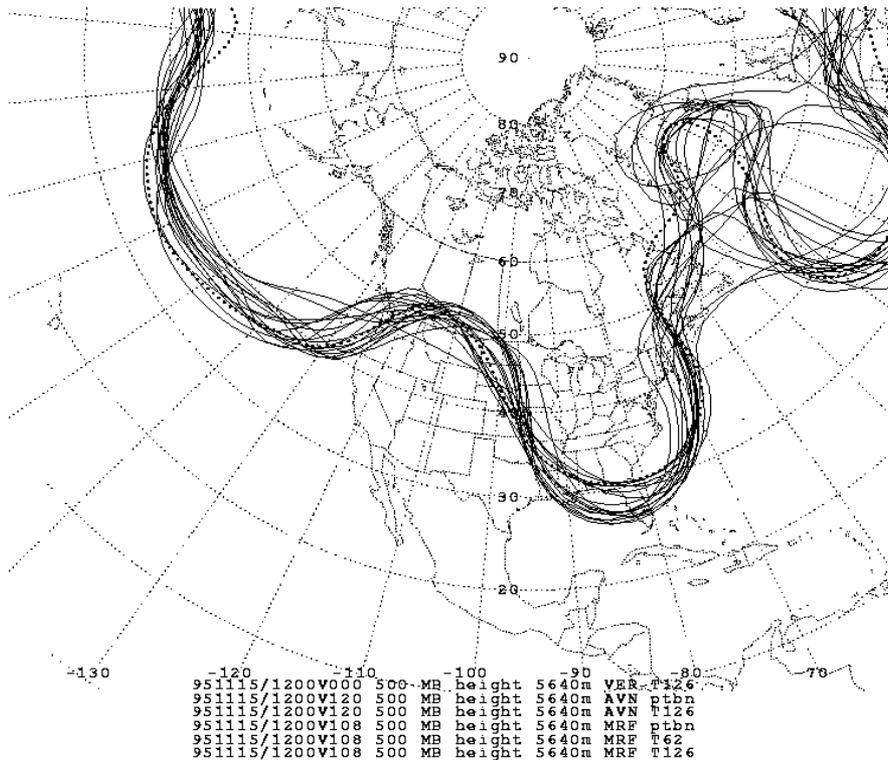
Example of a **very predictable 6-day forecast**, with “errors of the day”



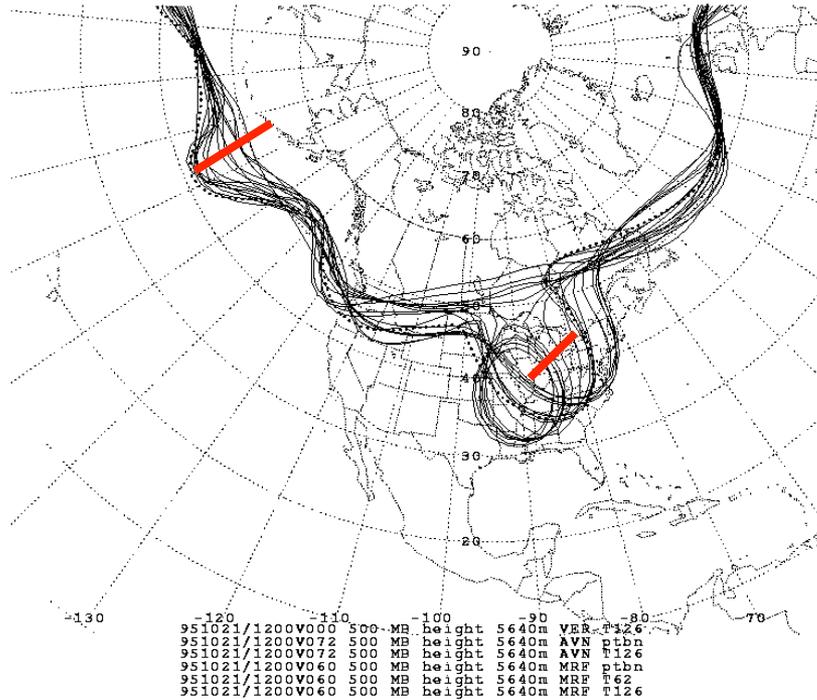
**This forecast led to the first 5-day prediction of a blizzard!**

**The errors of the day are instabilities of the background flow. At the same verification time, the forecast uncertainties have *the same shape***

4 days and 6 days ensemble forecasts verifying on 15 Nov 1995



# Strong instabilities of the background tend to have simple shapes: perturbations lie in a low-dimensional subspace of bred vectors



2.5 day forecast verifying  
on 95/10/21.

Note that the bred vectors  
(difference between the  
forecasts) lie on a 1-D space

This allows EnKF with 50,  
not millions of forecasts!

## Ensemble forecasting allowed public forecasts to go from 3 days to 7 days!

# In the rest of this talk, we will look at chaos in coupled fast-slow systems

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- The atmosphere has fast (e.g., convective clouds, 20 min) and slow instabilities (e.g., baroclinic or weather instabilities 3-7 days)
- The coupled ocean-atmosphere system has even slower instabilities (El Niño-Southern Oscillation, 3-7 years)
- In order to predict these phenomena, we need to isolate fast and slow instabilities
- If we can predict ENSO, we can predict climate anomalies a year or more in advance

# We coupled slow and a fast Lorenz (1963) 3-variable models (Peña and Kalnay, 2004)

Fast equations

$$\frac{dx_1}{dt} = \sigma(y_1 - x_1) - C_1(Sx_2 + O)$$

$$\frac{dy_1}{dt} = rx_1 - y_1 - x_1z_1 + C_1(Sy_2 + O)$$

$$\frac{dz_1}{dt} = x_1y_1 - bz_1 + C_1(Sz_2)$$

Slow equations

$$\frac{1}{\tau} \frac{dx_2}{dt} = \sigma(y_2 - x_2) - C_2(x_1 + O)$$

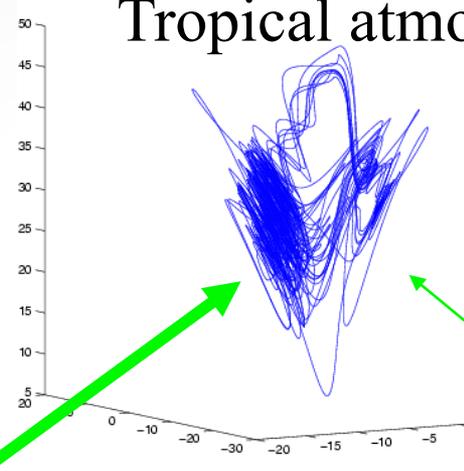
$$\frac{1}{\tau} \frac{dy_2}{dt} = rx_2 - y_2 - Sx_2z_2 + C_2(y_1 + O)$$

$$\frac{1}{\tau} \frac{dz_2}{dt} = Sx_2y_2 - bz_2 + C_2(z_1)$$

“**Tropical-extratropical**” (triply-coupled) system: the ENSO tropical atmosphere is weakly coupled to a **fast “extratropical atmosphere”** with weather noise

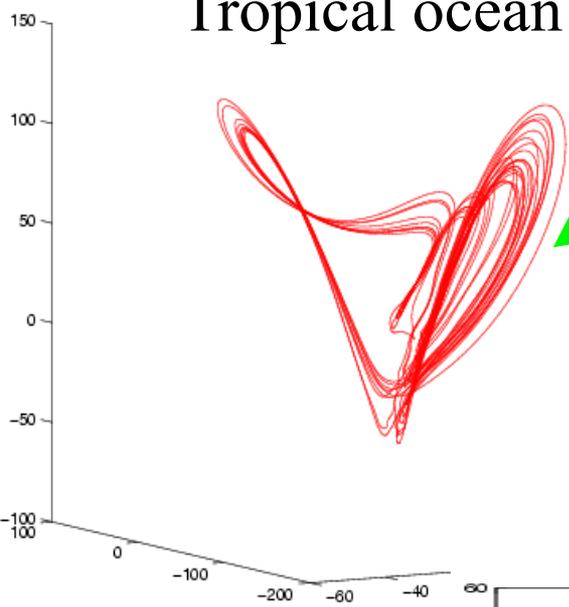
# Tropical atmosphere

Solution of the "tropical atmosphere" system



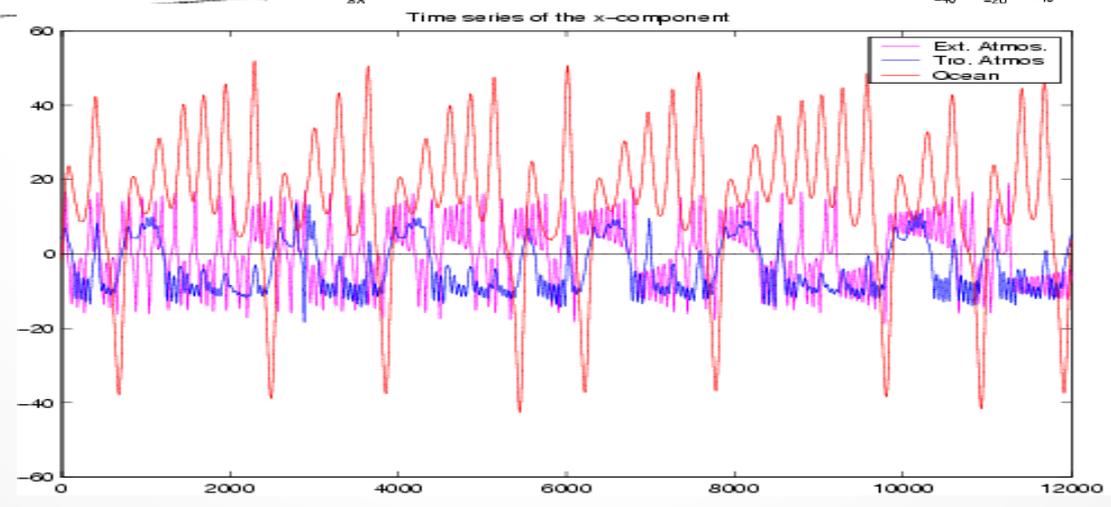
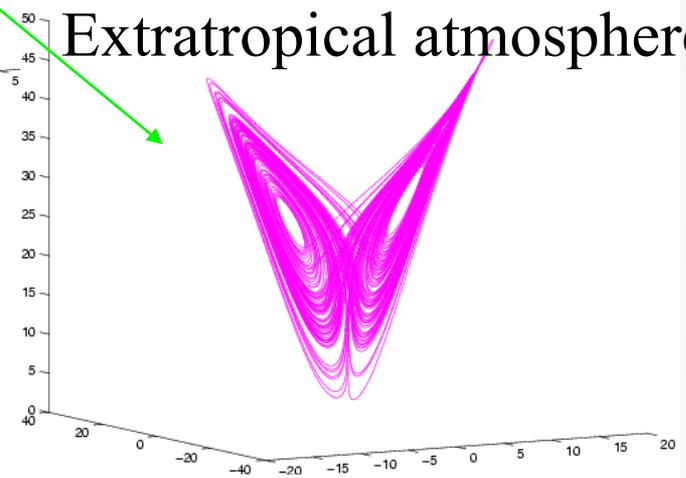
# Tropical ocean

Solution of the "ocean" system



# Extratropical atmosphere

Solution of the "extratropical atmosphere" system



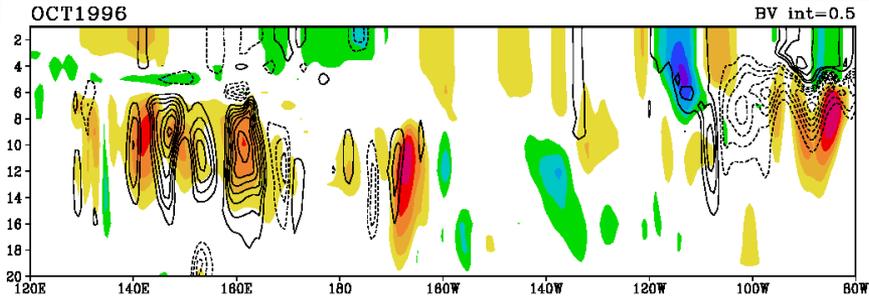
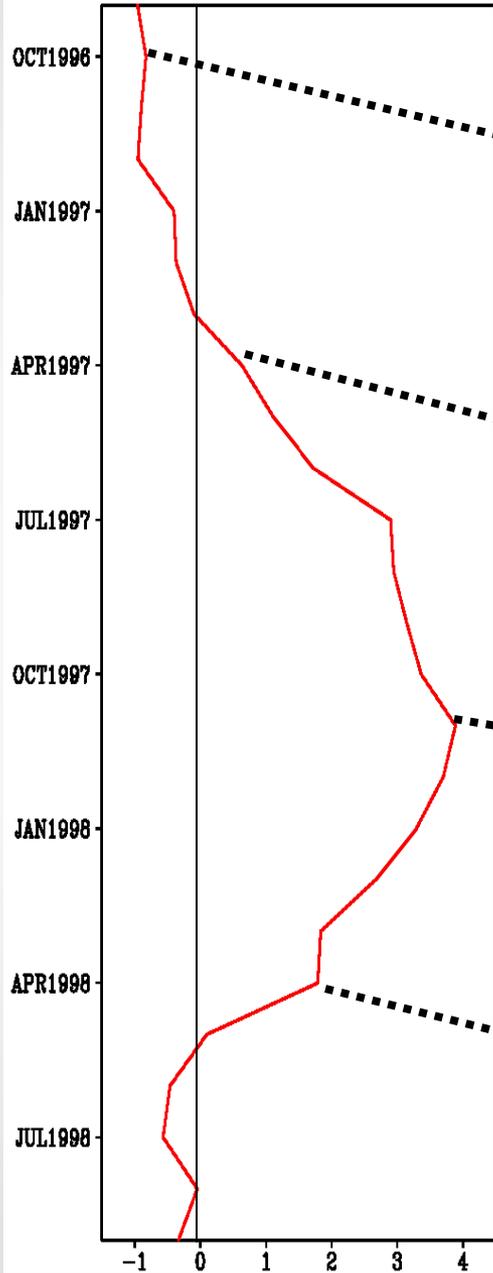
# **We showed that it is possible to separate fast and slow instabilities**

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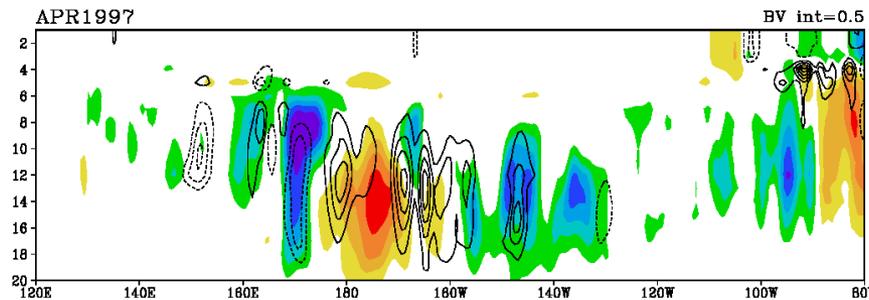
- That allowed to find the shape of El Niño instabilities
- The instabilities (Bred Vectors) should have the same shape as the El Niño forecast errors.

# Niño3 index

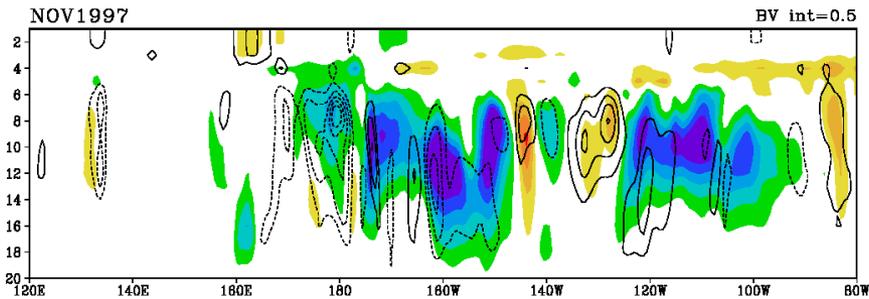
## Yang (2005): Vertical cross-section at Equator for BV (contours) and 1 month forecast error (color)



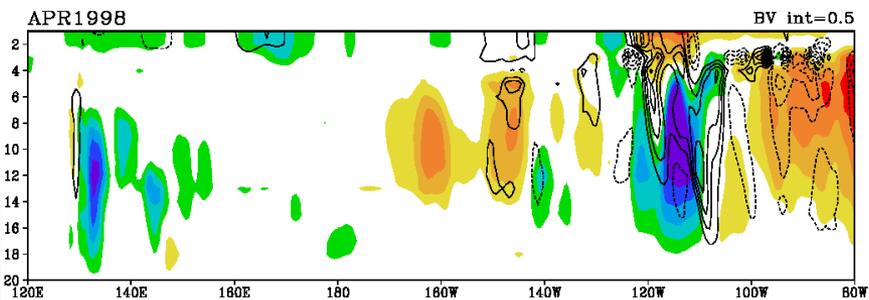
Before 97' El Niño, error is located in W. Pacific and near coast region



During development, error shifts to lower levels of C. Pacific.

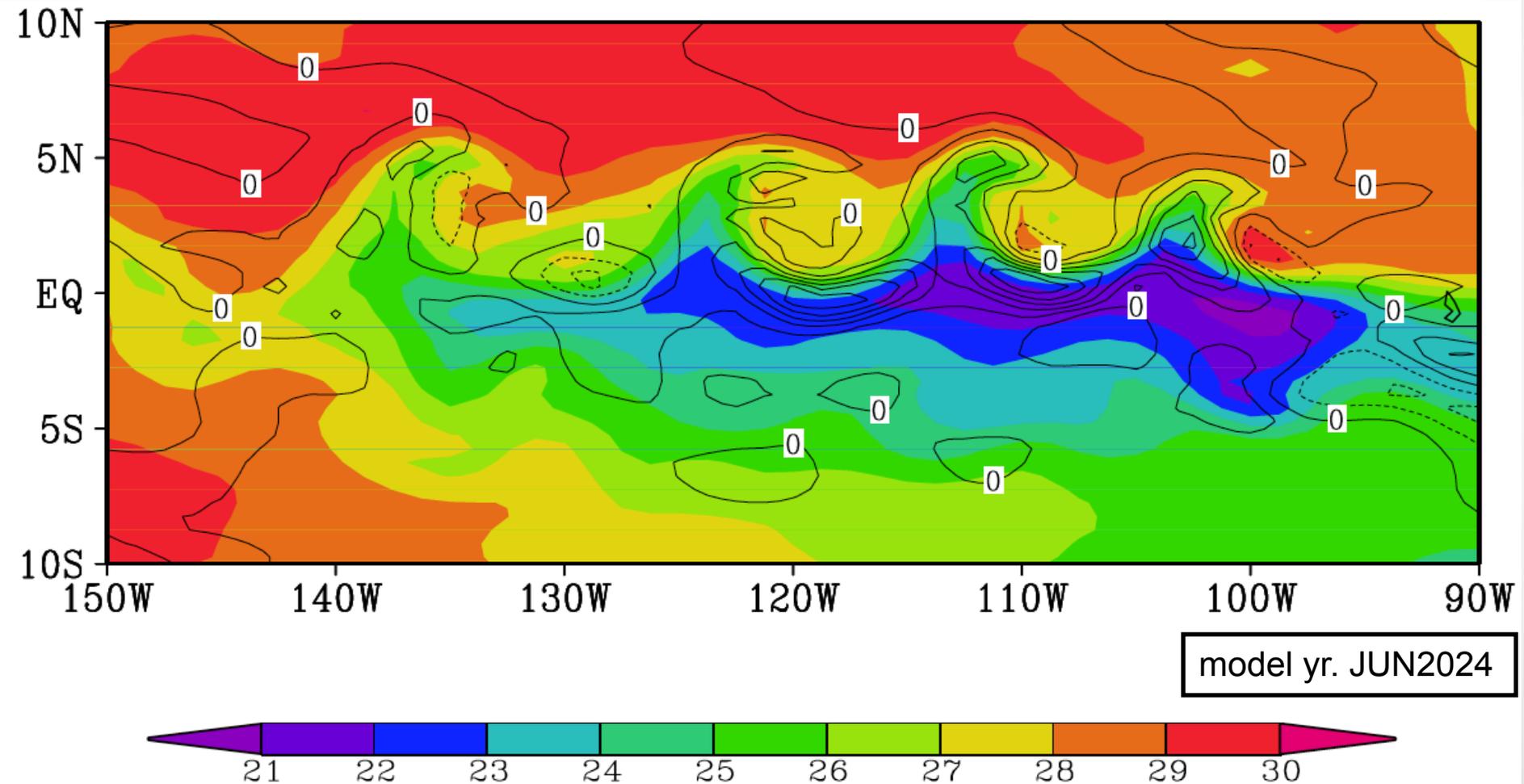


At mature stage, error shifts further east and it is smallest near the coast.



After the event, error is located mostly in E. Pacific.

# Yang et al., 2006: Bred Vectors (contours) overlay Tropical Instability waves (SST, shades): make them grow and break!

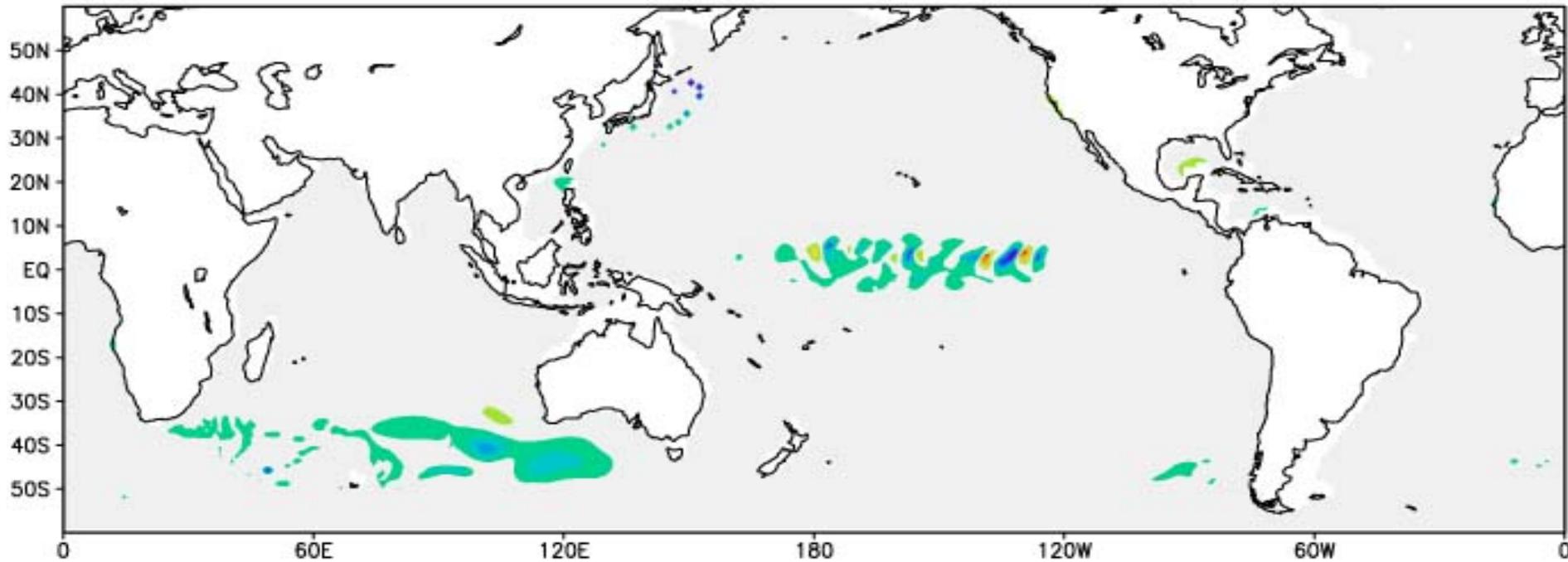


Michael Ghil: "Look at the energy of the bred vectors"

# Hoffman et al (2008): finding ocean instabilities with breeding time-scale 10-days captures tropical instabilities

Breeding time scale: 10 days

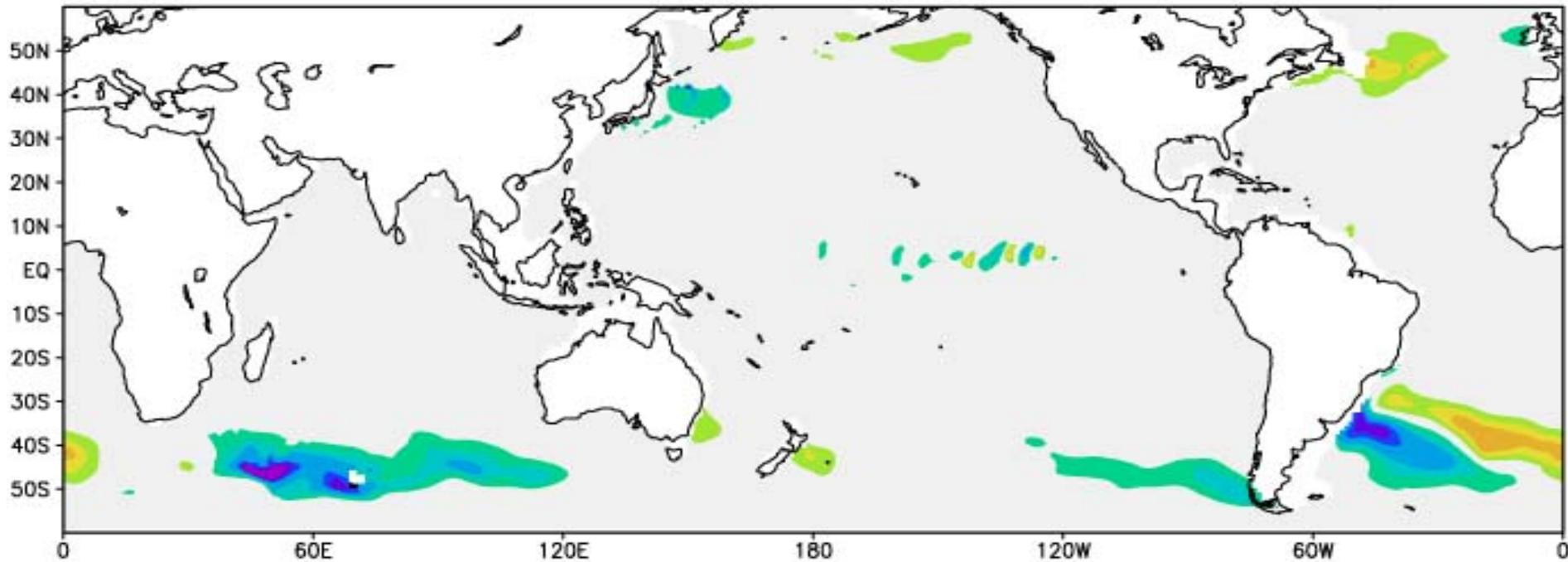
SST Bred Vector on December 1, 1988



-0.3 -0.25 -0.2 -0.15 -0.1 -0.05 0.05 0.1 0.15 0.2 0.25 0.3

# When the rescaling time scale is 30 days, extratropical instabilities dominate

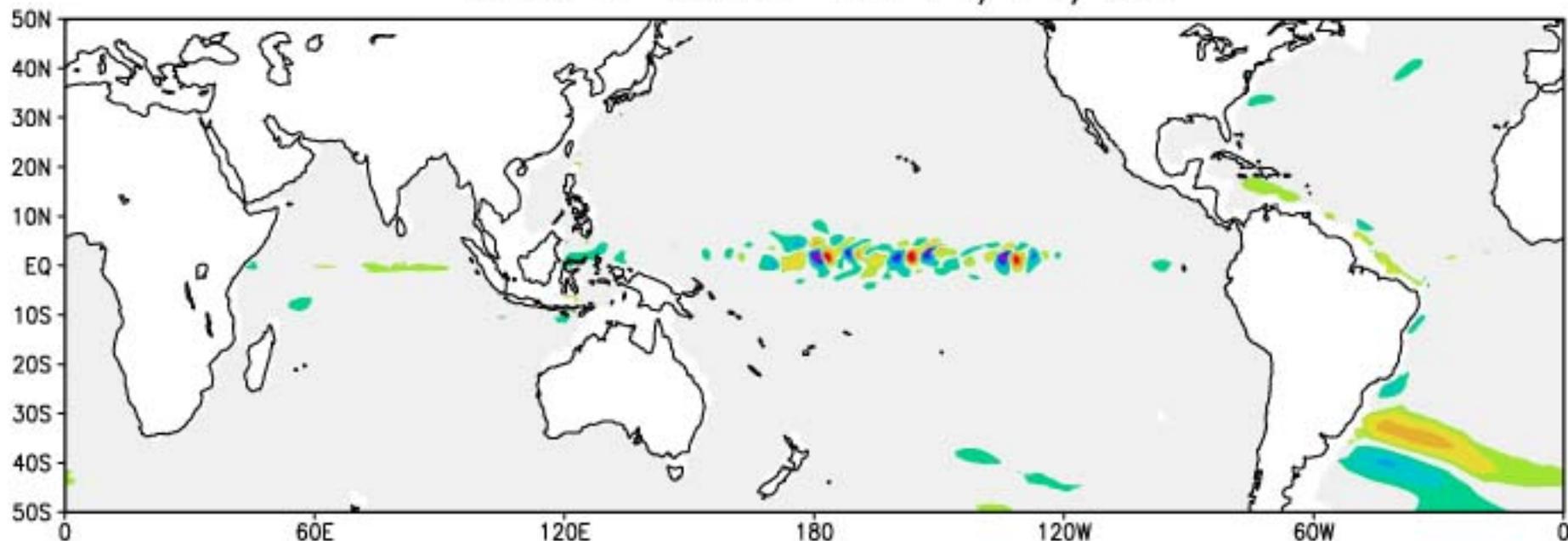
SST Bred Vector on December 11, 1988  
30 Day Rescaling Time, 0.2 Rescaling Factor



-0.7 -0.6 -0.5 -0.3 -0.2 -0.1 0.1 0.2 0.3 0.5 0.6 0.7

**Here we have both Tropical and  
“South Atlantic Convergence Zone” instabilities.  
Can we determine the dynamic origin of these instabilities? Yes!**

Bred U Vector on 11/11/88

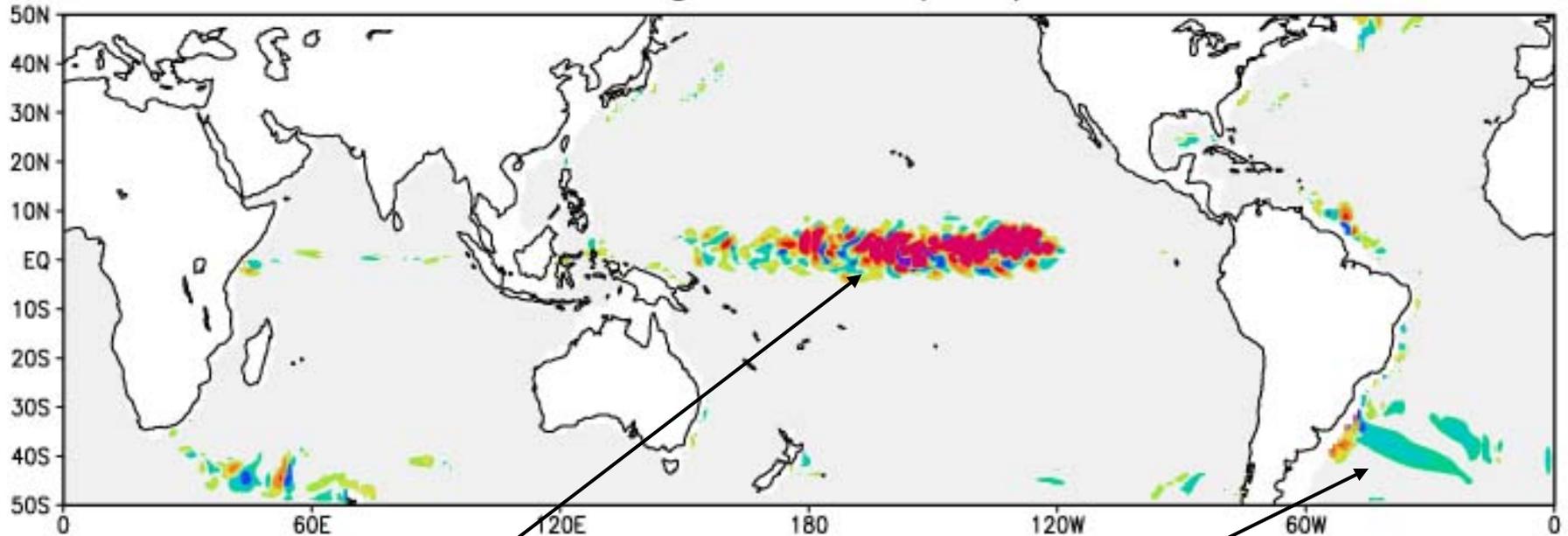


The Bred Vector Kinetic Energy equation can be computed exactly because both control and perturbed solutions satisfy the model equations!

$$\frac{\partial KE_{bv}}{\partial t} = \text{horizontal fluxes} - \rho_b g w_b + \dots$$

Conversion from potential to kinetic energy

-rho'\*g\*w' at 11/11/88



PE → KE

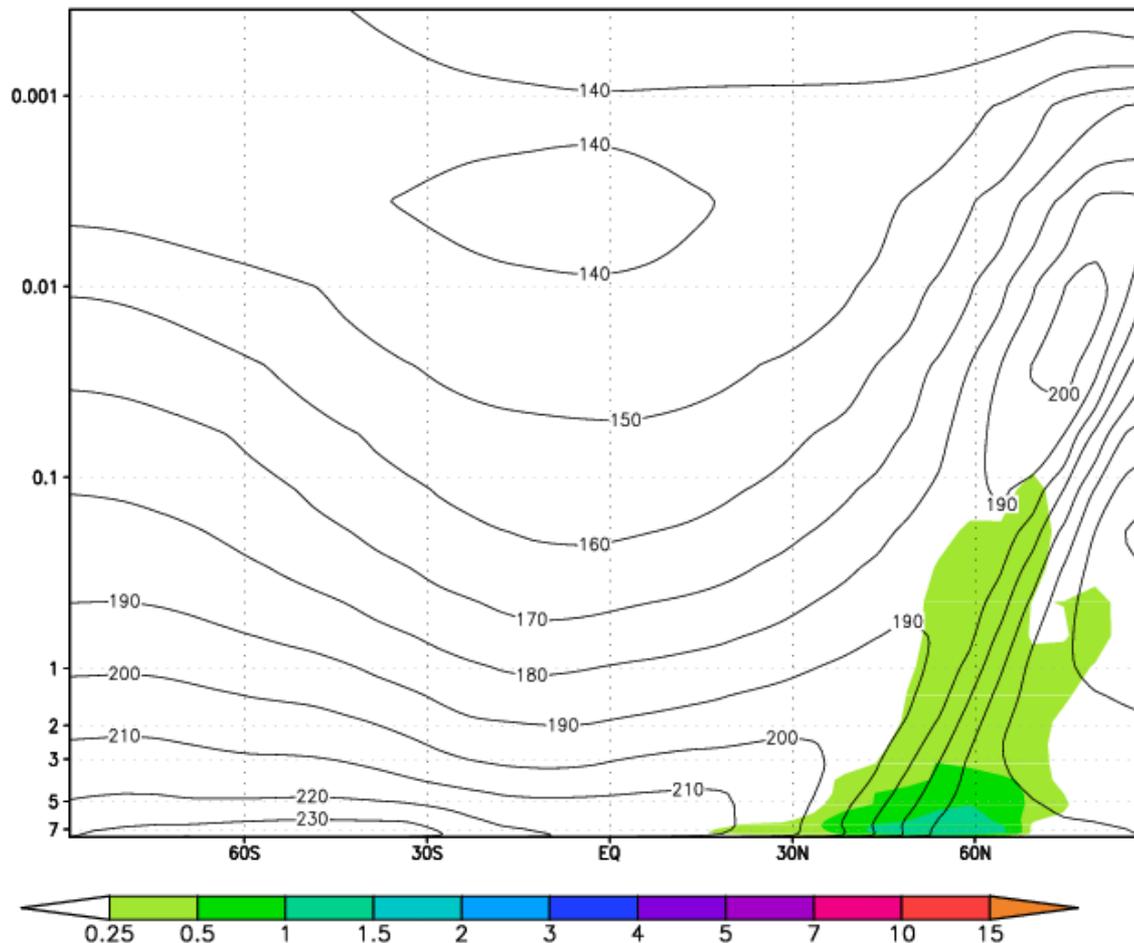
KE → PE!





# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 078 Hour 00



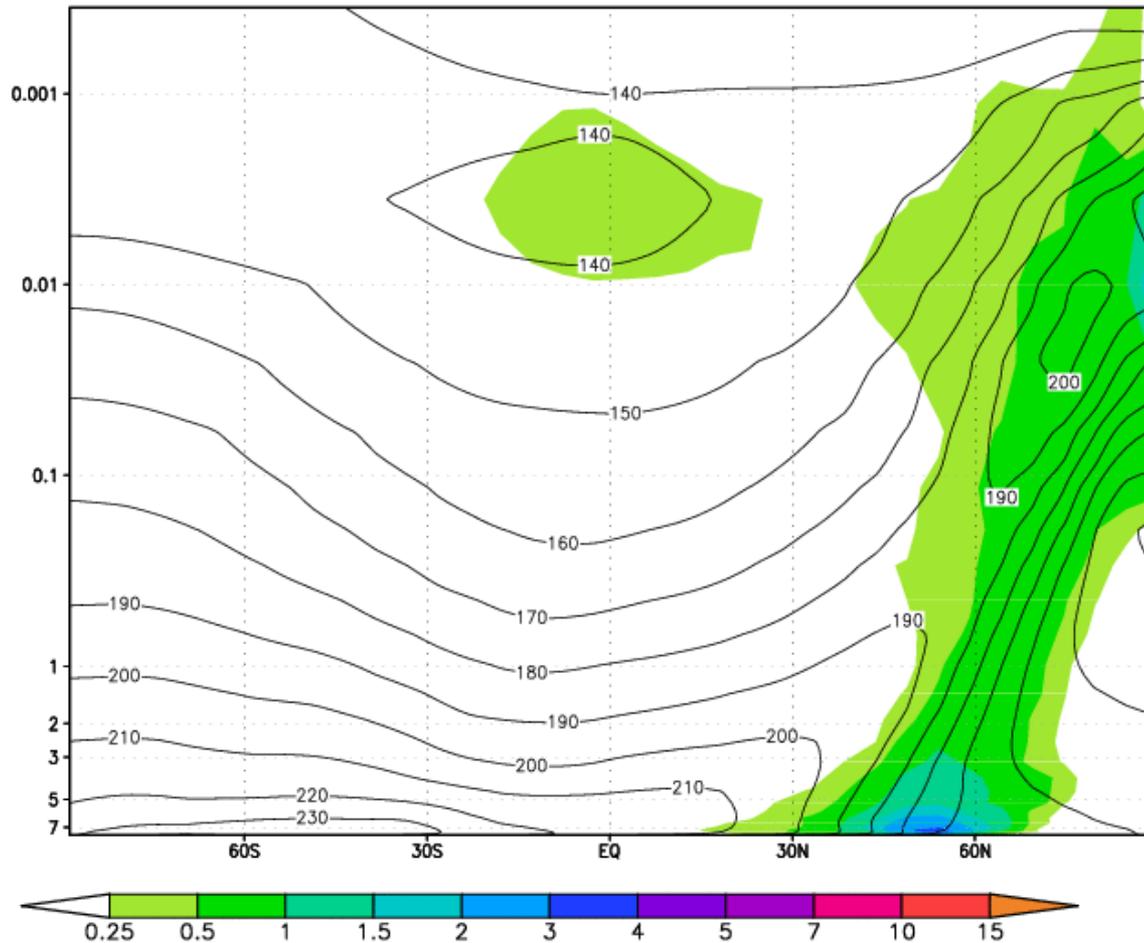
Day 078 (Ls= 302, Boreal Winter): BV activity near surface temperature front begins to flare up.

**Greybush et al., 2011**



# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 080 Hour 00

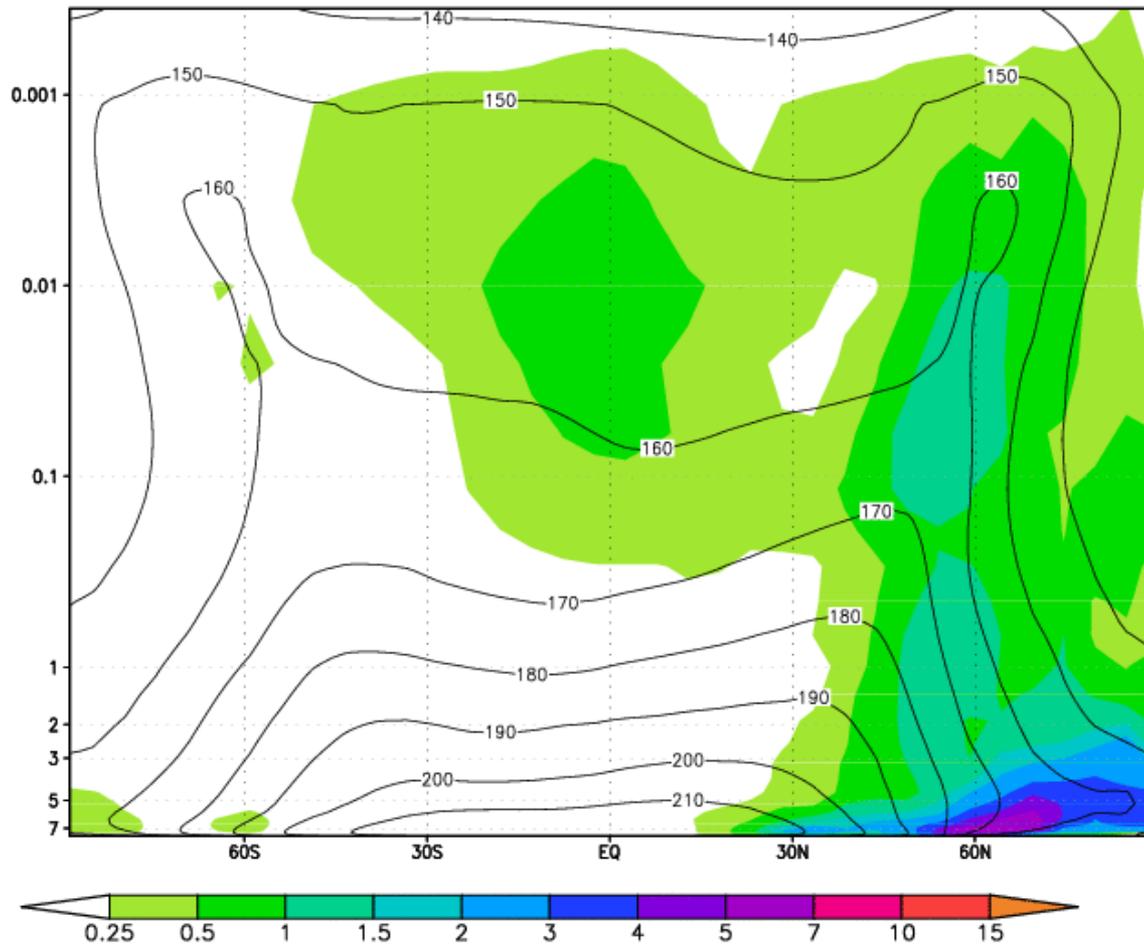


Day 080 (Ls= 304, Boreal Winter): Just two days later, BV now extends vertically along the length of the front. Connection to the upper level tropics begins.



# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 175 Hour 00

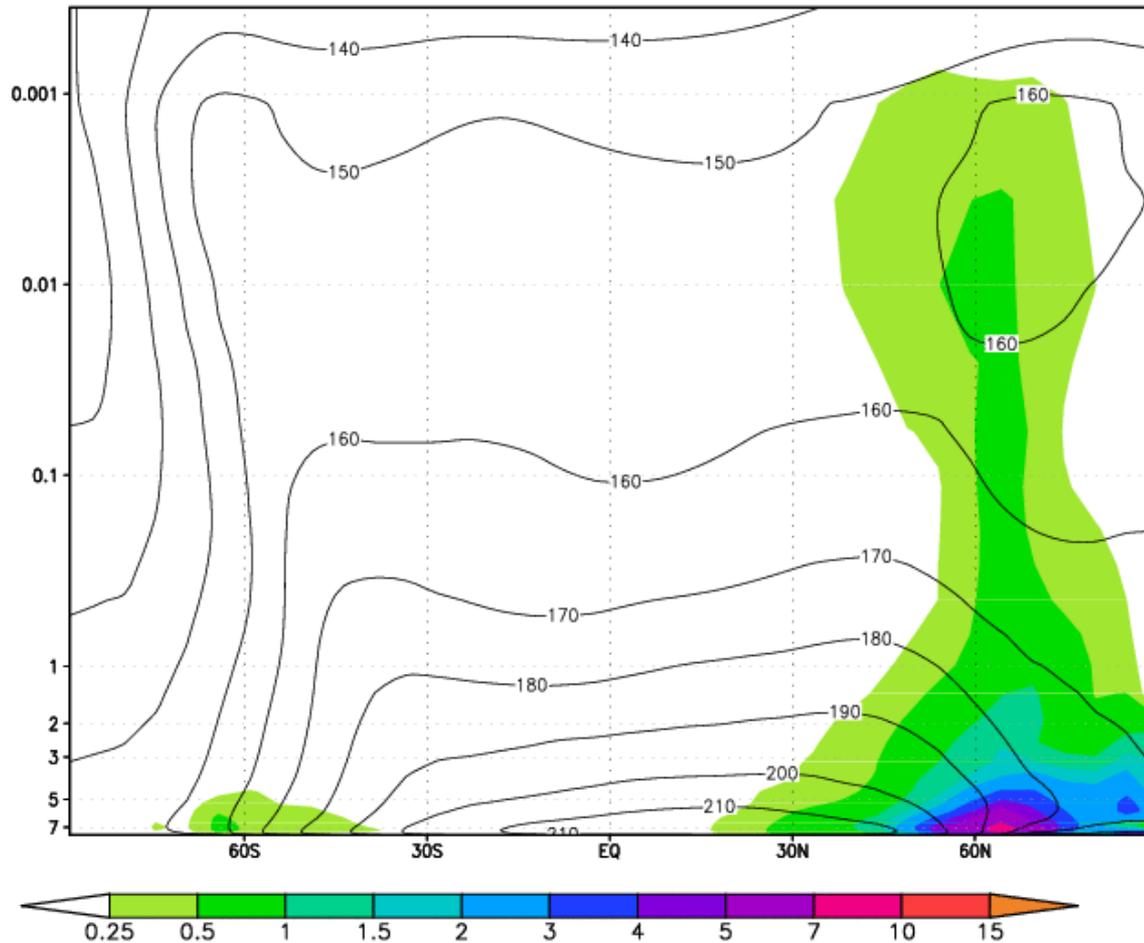


Day 175 (Ls= 358, near boreal vernal equinox): Typical winter BV activity along temperature front with upper level tropical connection. First hint of southern hemisphere activity.



# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 235 Hour 00

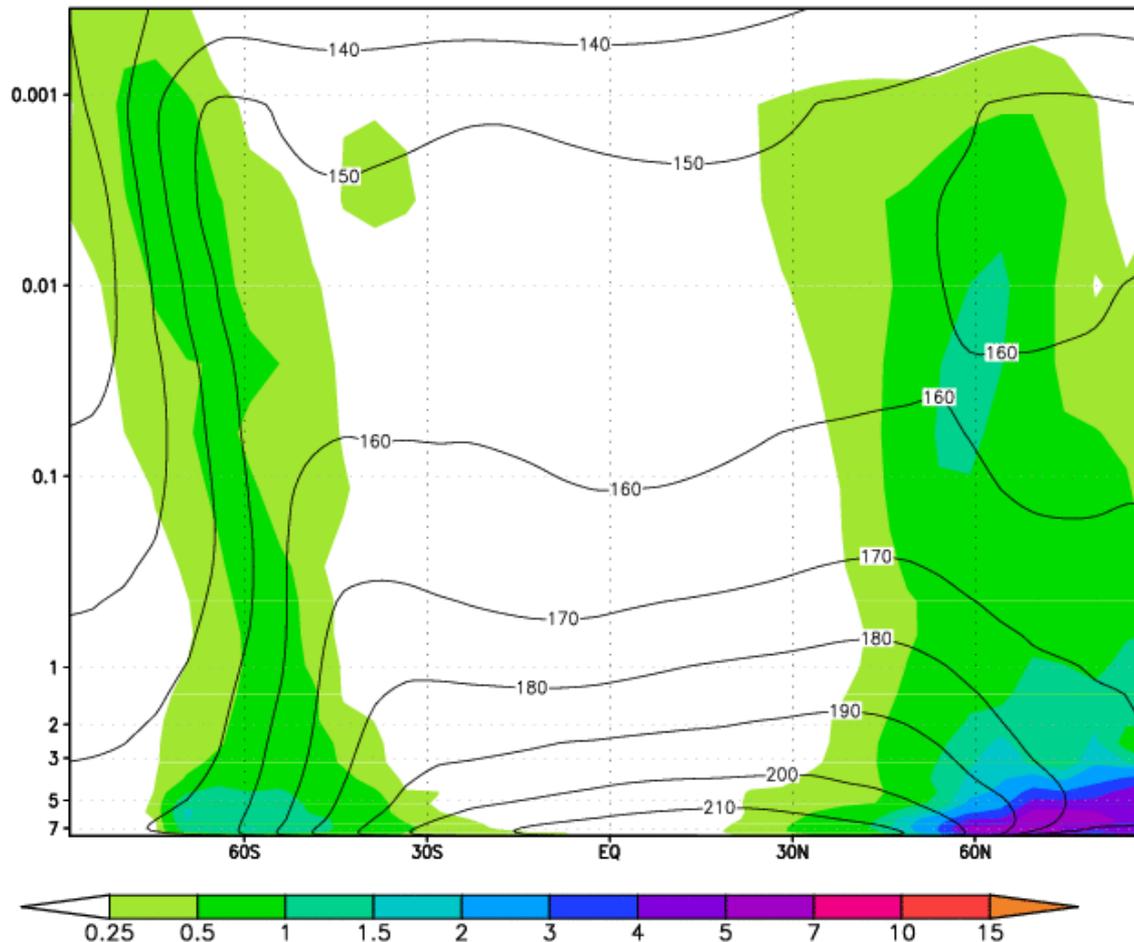


Day 235 (Ls= 22, early boreal spring): Winter BV activity has begun to weaken, as the tropical connection has disappeared.



# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 240 Hour 00

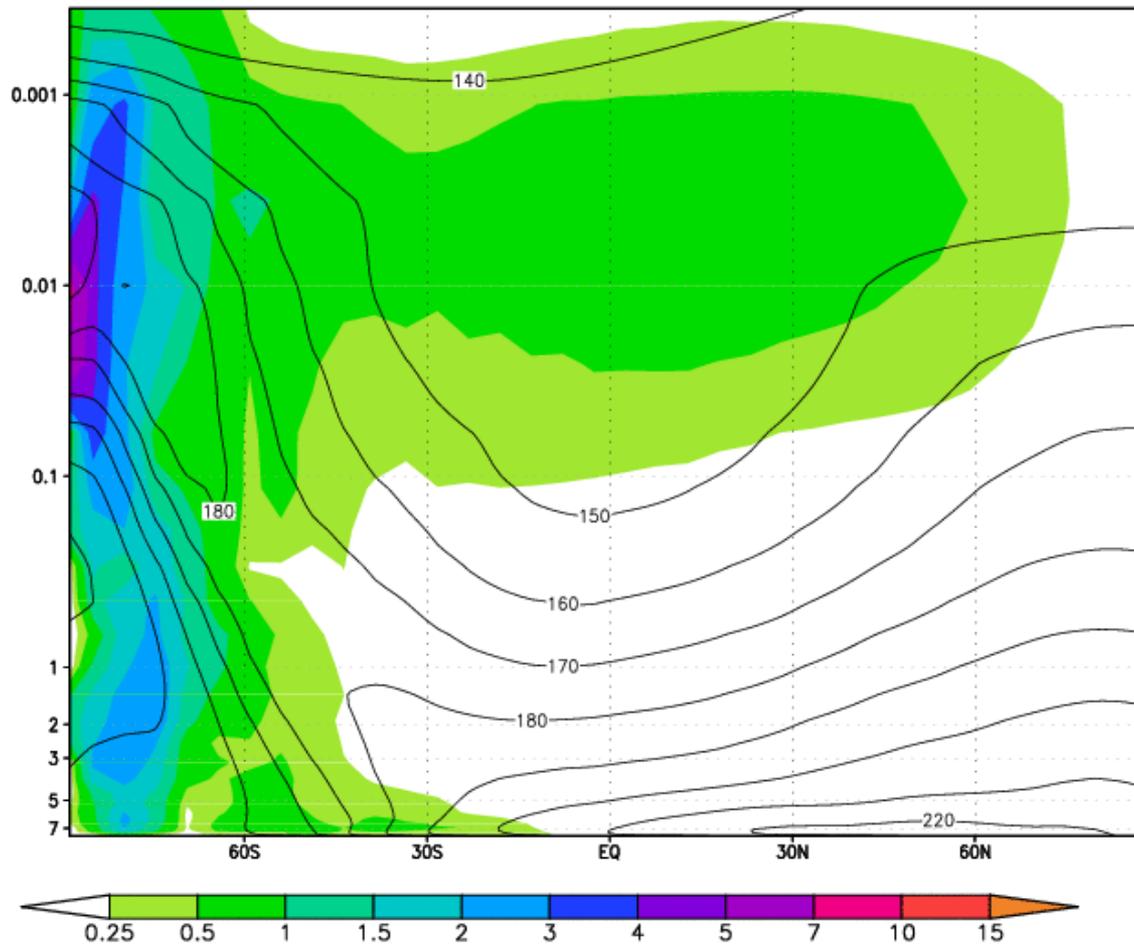


Day 240 (Ls= 230, early boreal spring): Southern hemisphere activity has now grown rapidly along austral front.



# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 430 Hour 00

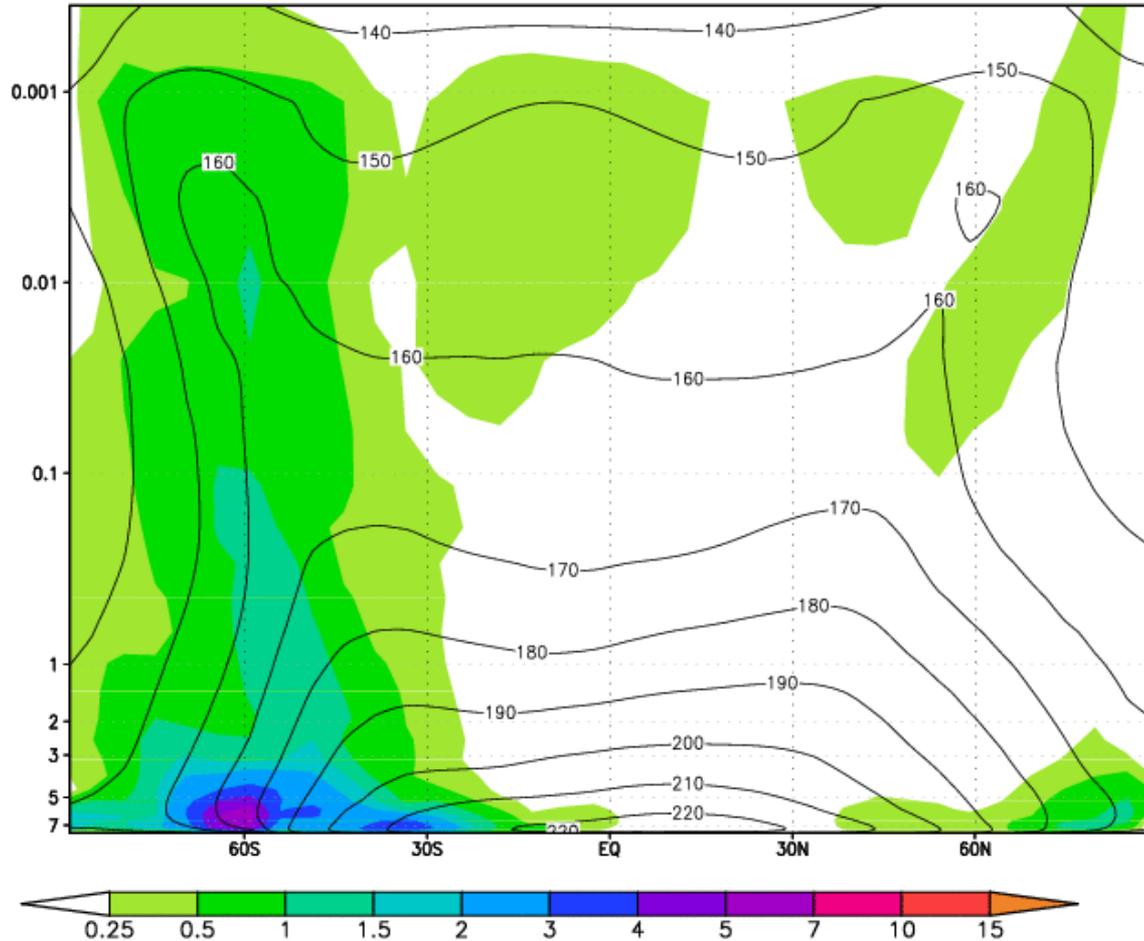


Day 430 (Ls= 116. austral mid-winter): Southern hemisphere BV activity now assumes full spatial extent.



# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 549 Hour 00

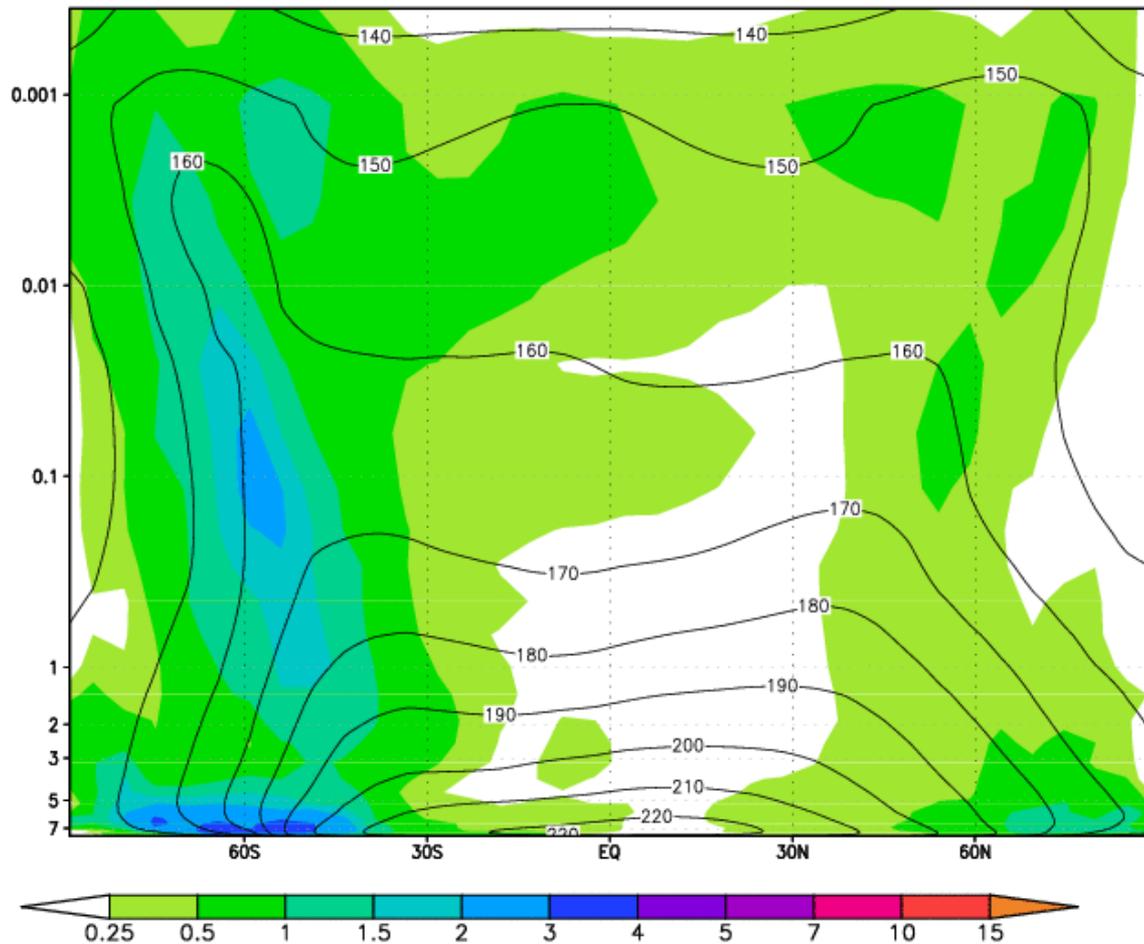


Day 549 (near boreal autumn equinox): Signs BV of activity in the northern hemisphere have resumed.



# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 551 Hour 00

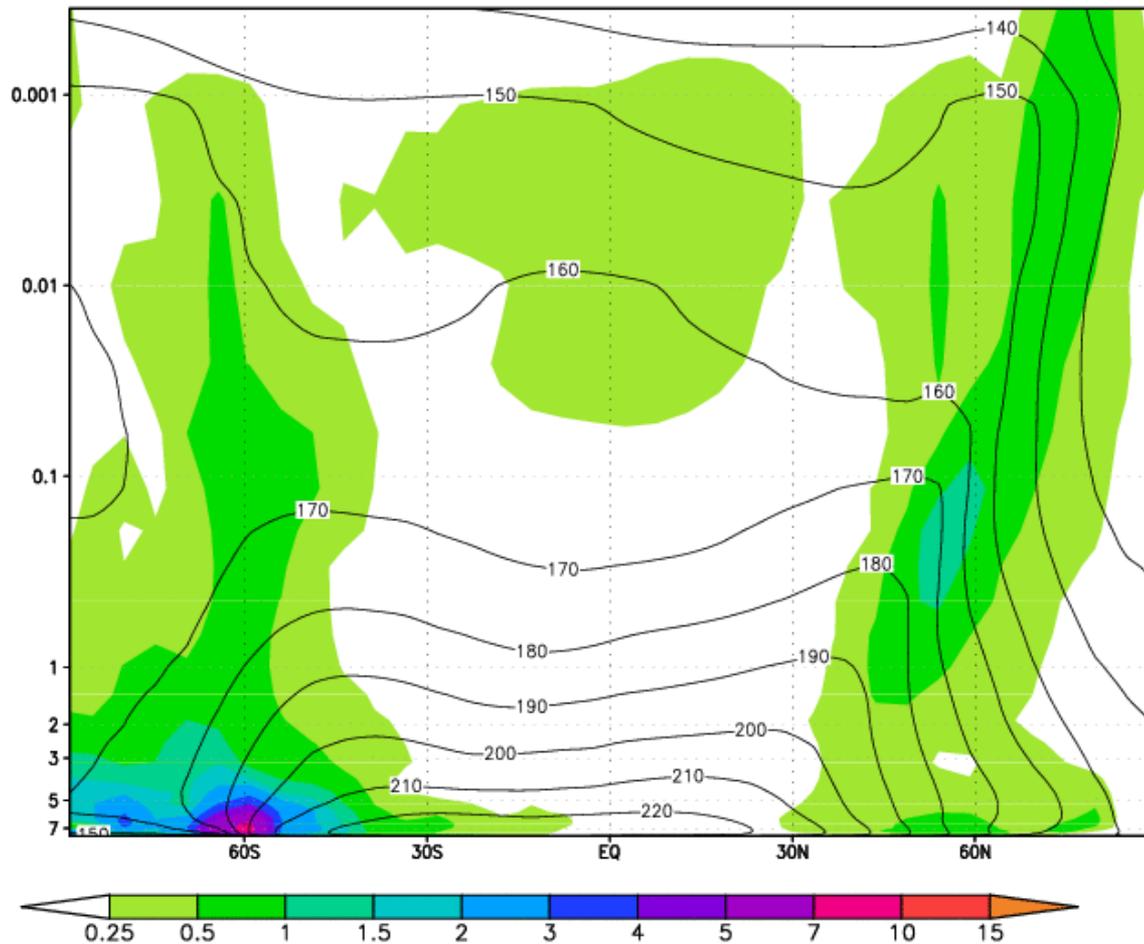


Day 551 (Ls= 180, boreal autumn equinox): Activity in northern hemisphere has extended vertically.



# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 590 Hour 00

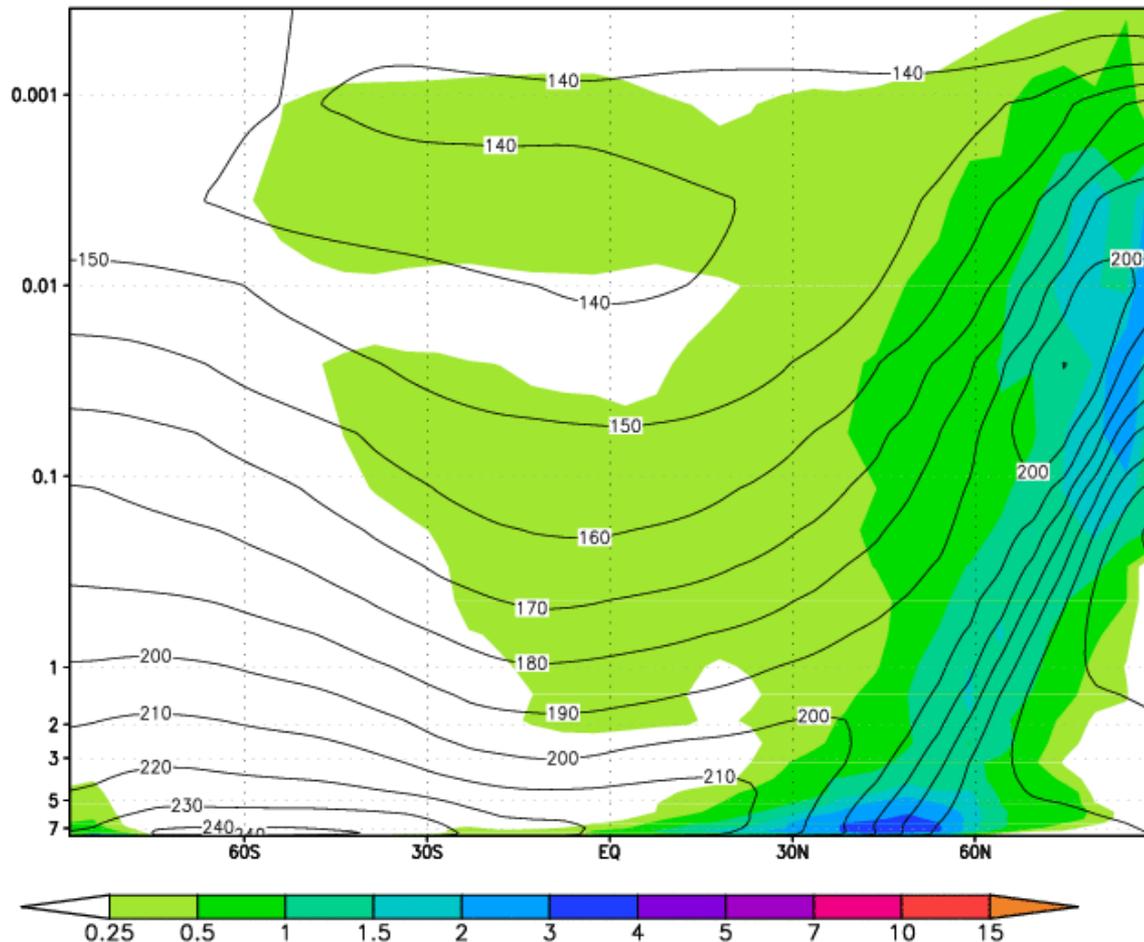


Day 590 (mid boreal autumn): Activity in both hemispheres, but most intense along southern polar front.



# MGCM-LETKF-**TES** Martian Atmosphere Reanalysis Project

Martian Bred Vector Plot – Temperature [K] Zonal Mean RMS BV Magnitude – Day 668 Hour 00



Day 668 (Ls= 252, prior to boreal winter solstice): The seasons have returned full circle, with southern hemisphere activity fading and northern winter dominant.

# Summary

- We can apply the same uncertainty methods to the 3-variable Lorenz model and to huge models.
- The Bred Vectors are the instabilities that dominate the forecast errors: we use them as initial perturbations for ensemble forecasting.
- They just require running the model twice, and are low dimensional.
- Ensemble forecasting allowed the NWS to increase the public predictions from 3 to 7 days and deal with the uncertainty.
- In the “unpredictable” Lorenz model the growth of Bred Vectors predicts the changes of regime and how long they will last.
- Breeding and the LETKF can be applied to a time series even without knowing the model.
- Bred Vectors help explain the physical origin of instabilities in the ocean and in Mars.
- LETKF (related to breeding) is the ultimate method to explore and “beat chaos” through data assimilation