The device physics we need to build thermodynamic computers

Suhas Kumar

Hewlett Packard Labs

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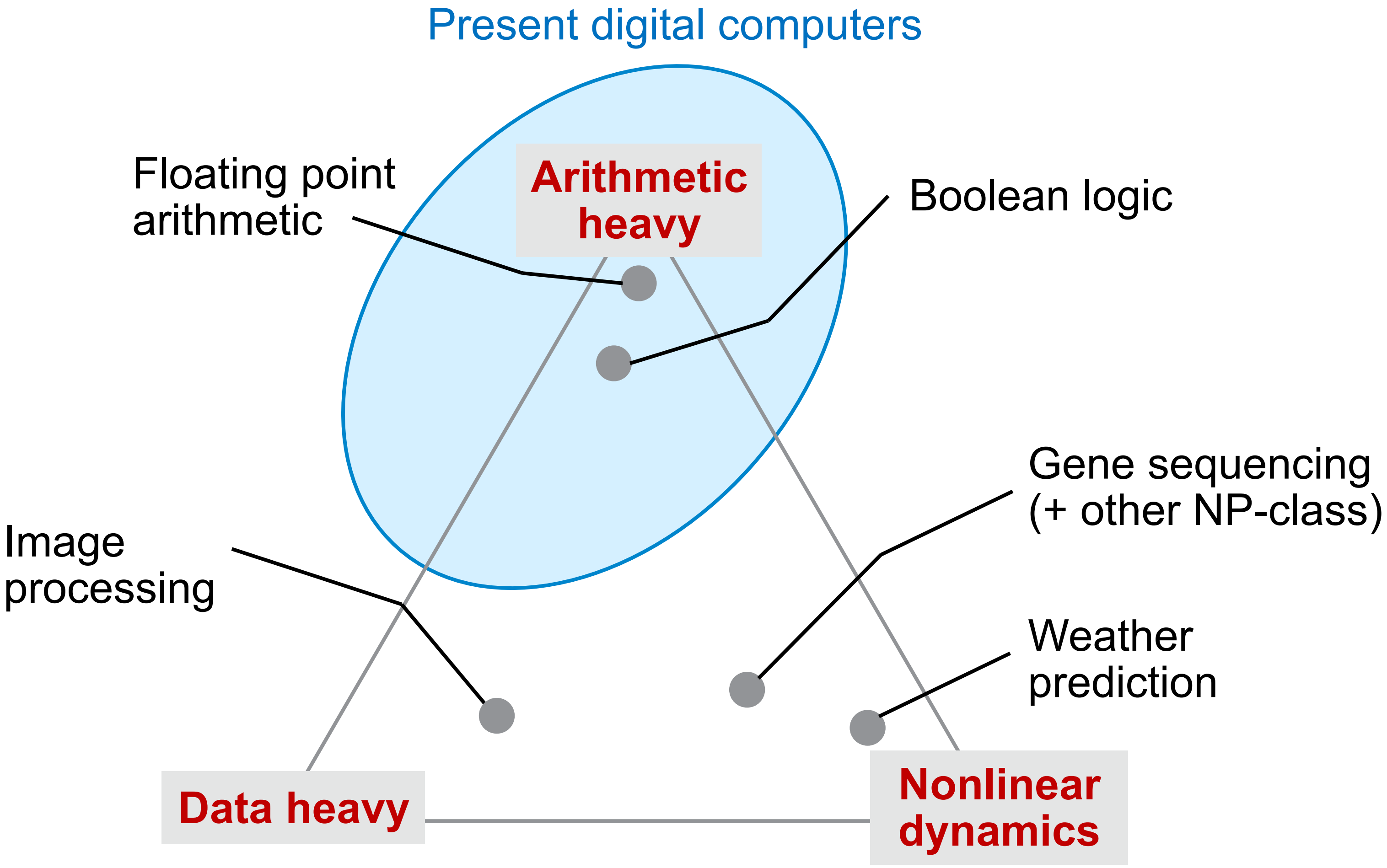
The voids of computing

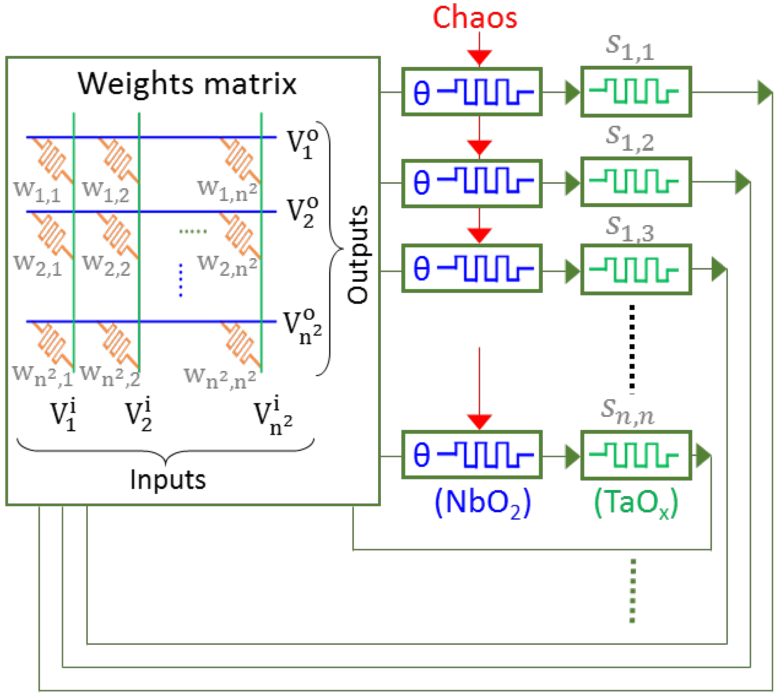


Major limitations of digital computers:

- End of Moore's law

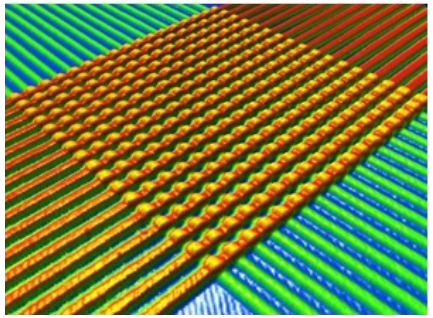
- Von Neumann architecture
- Boltzmann tyranny
- Boolean logic
- Turing limit

A transistorless all-“memristor” Hopfield network

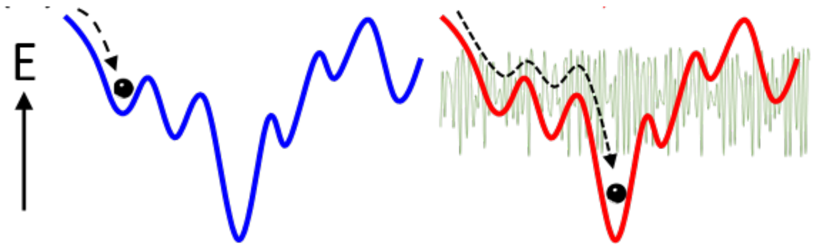


Synapses + neurons

1. Synaptic memristors – nonvolatile storage
   - New device property: analog tunability
2. Neuronic memristors – volatile storage + nonlinearity
   - New device property: chaotic dynamics



Memristor matrix



Noise-driven annealing.

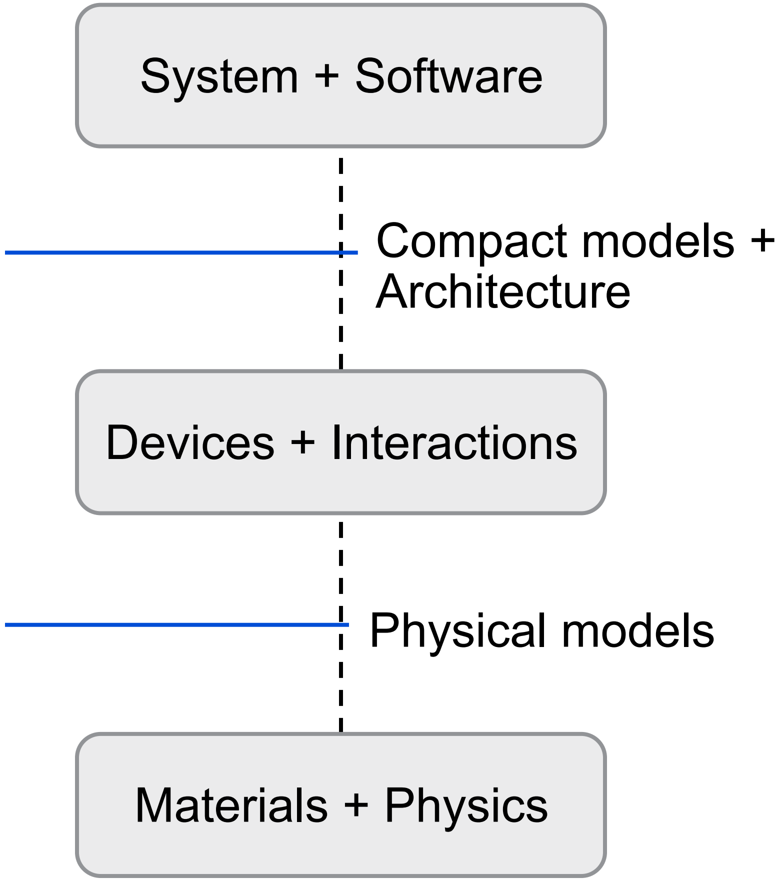
Kumar *et al.*, *Nature*, 548, 318 (2017)

Performance benchmarking – larger NP-hard problems

	memristor-HNN	GPU	FPGA	D-wave 2000Q
Clock frequency	1 GHz	1.582 GHz	0.1 GHz	
Time to solution (TTS)	$0.3\,\mu s$	$10\,\mu s$	$> 1\,\mu s$	10^4 s
Power	3.48 W	< 250 W	70 W	$25,000\,W$
Energy to solution	$0.96\,\mu J$	< 2.5 mJ		250 MJ
Solutions/s/Watts	1.04×10^6	< 2000	< 2000	4×10^{-9}
Connectivity	all-to-all	all-to-all	all-to-all	limited
Cryogenic cooling	No	No	No	Yes

Unpublished

A physics-driven computer program



New architectures, e.g.:
Hopfield networks,
Boltzmann machines

New device behaviors, e.g.:
Action potential, chaos

New physics, e.g.:
Thermal behavior during
Mott transitions

Nonlinear electronics – why it's important

All memristors are inherently nonlinear devices.

As devices are shrunk to the nanoscale, they interact with their environment → more state variables → nonlinearity is inevitable

$\eta \propto T\left(\frac{k_B}{C_{th}}\right)^{1/2} \frac{4\pi}{R_{th} C_{th}}$

Small devices can be driven by thermal noise, especially as they approach "kT".

How do we make use of this?

Why local activity is important

Nonlinearity →

Local activity →

Chaos →

Edge of chaos →

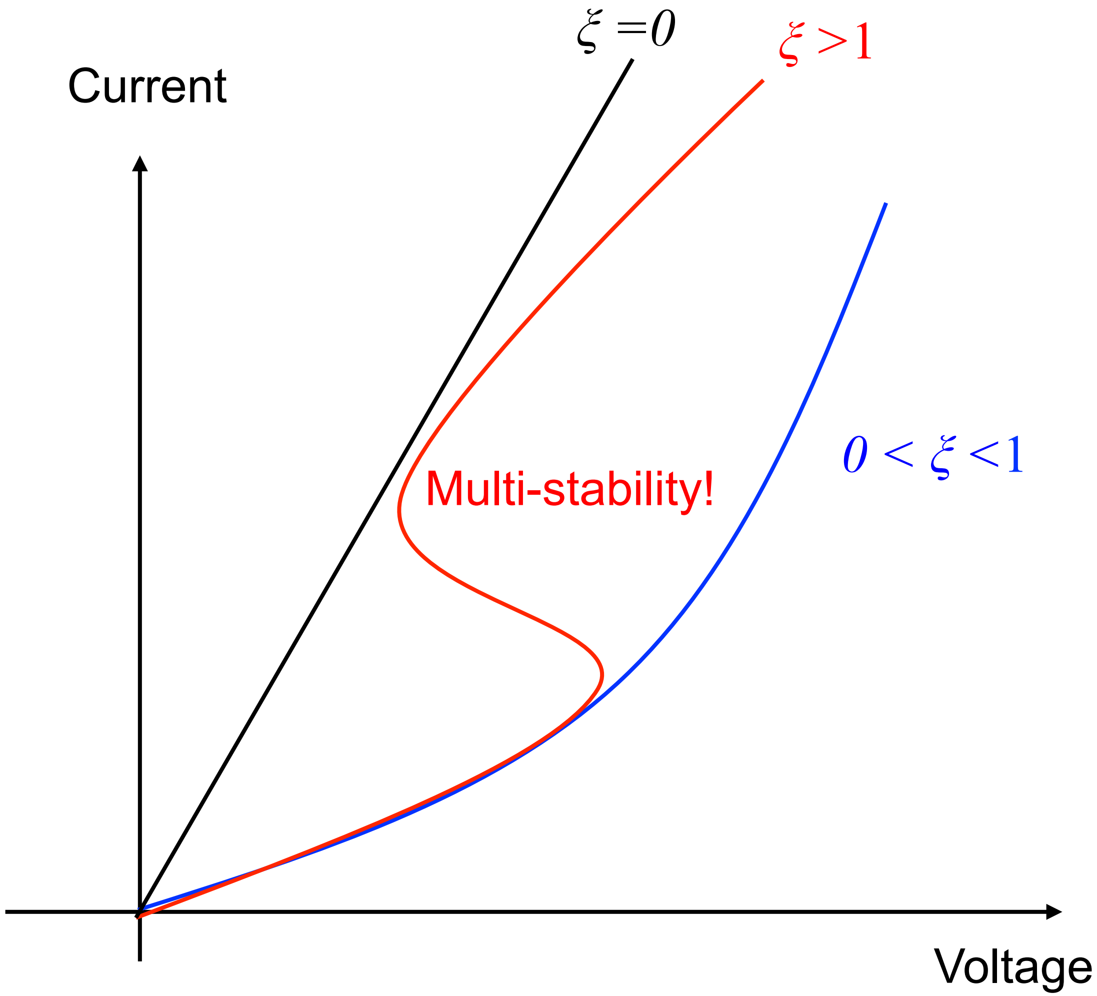
Complexity and emergence

Chua, "Local activity is the origin of complexity"
Chua, "Neurons are poised near the edge of chaos"
Chua, "Local activity principle"

Extreme nonlinearity?

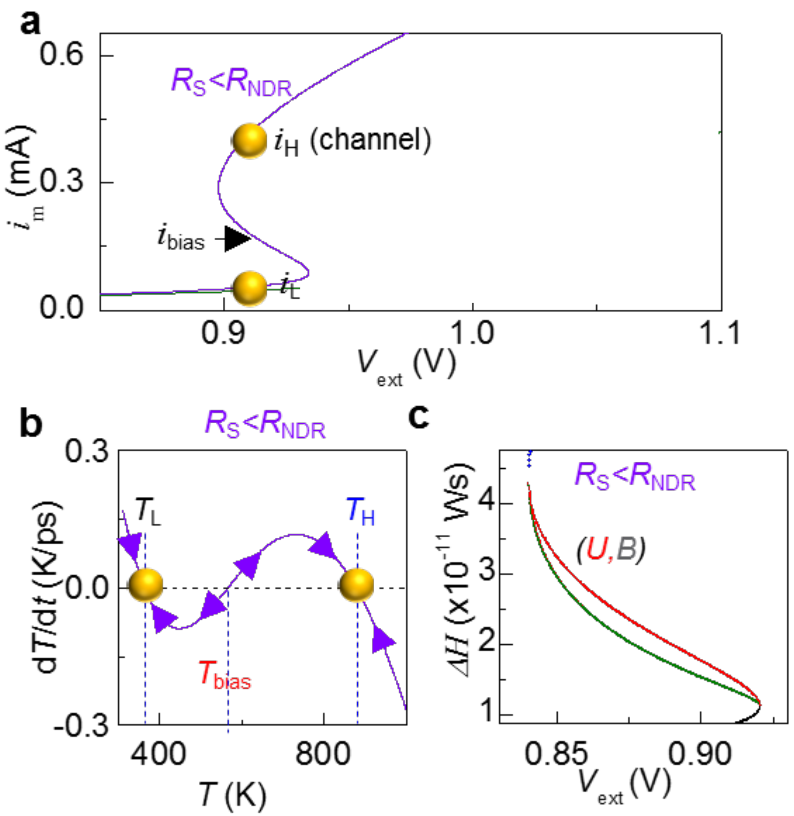
$i = Gv$

$G = \psi T \uparrow \xi$



Local activity → the ability to amplify energy

Thermodynamics of electronic devices: e.g.: decompositions



All device/circuit models so far:

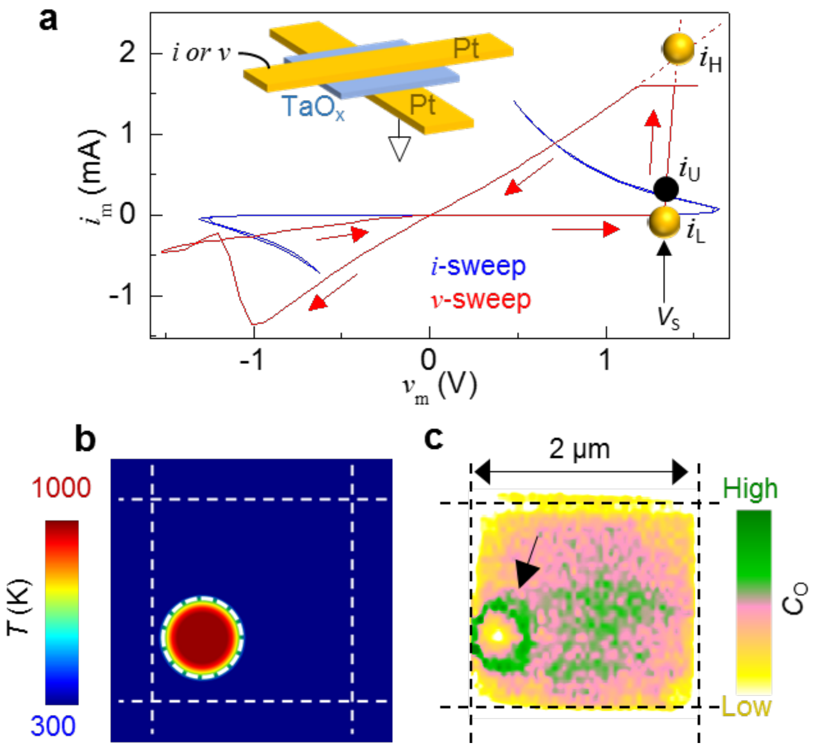
1. Behavior governed by *i, v, P, Q*

The two missing pieces of device models:

1. Behavior governed by thermodynamic quantities

2. Spontaneous symmetry breaking during instabilities.

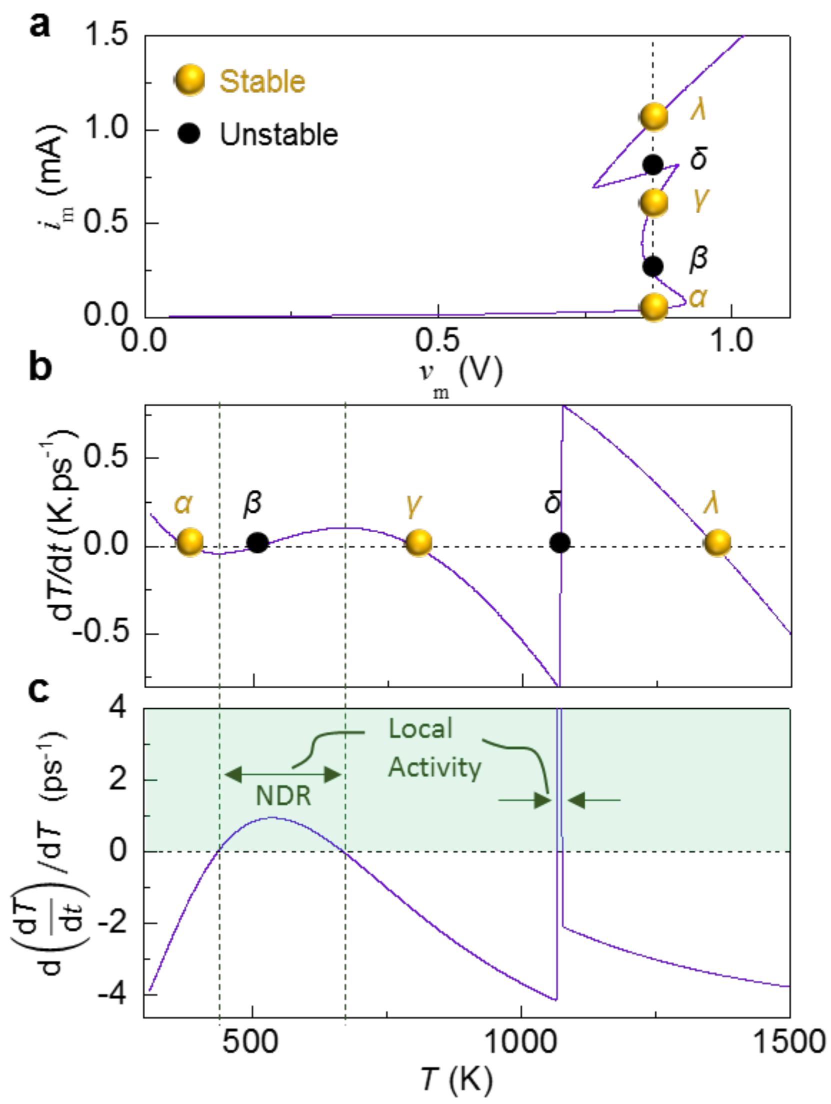
The origin of nonvolatile storage in ReRAM

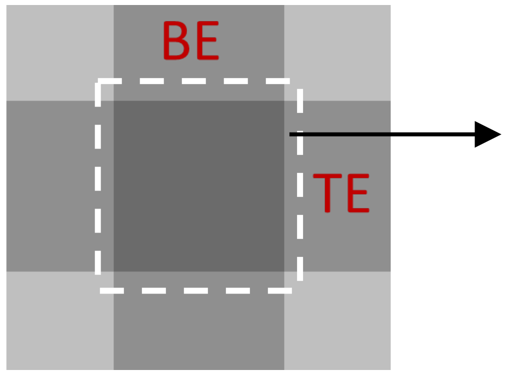


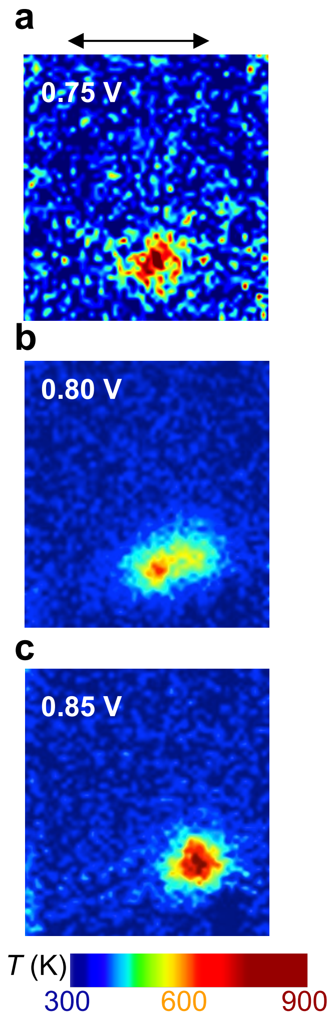
$$j_U = j_L \cdot (1-x) + j_H \cdot (x)$$

Kumar *et al.*, *Nature Communications,* (2018)
Kumar *et al., Advanced Materials,* 28, 2772 (2016)
Ridley, *Proc. Phys. Soc.,* 82, 954 (1963)

How thermal noise interacts with local activity







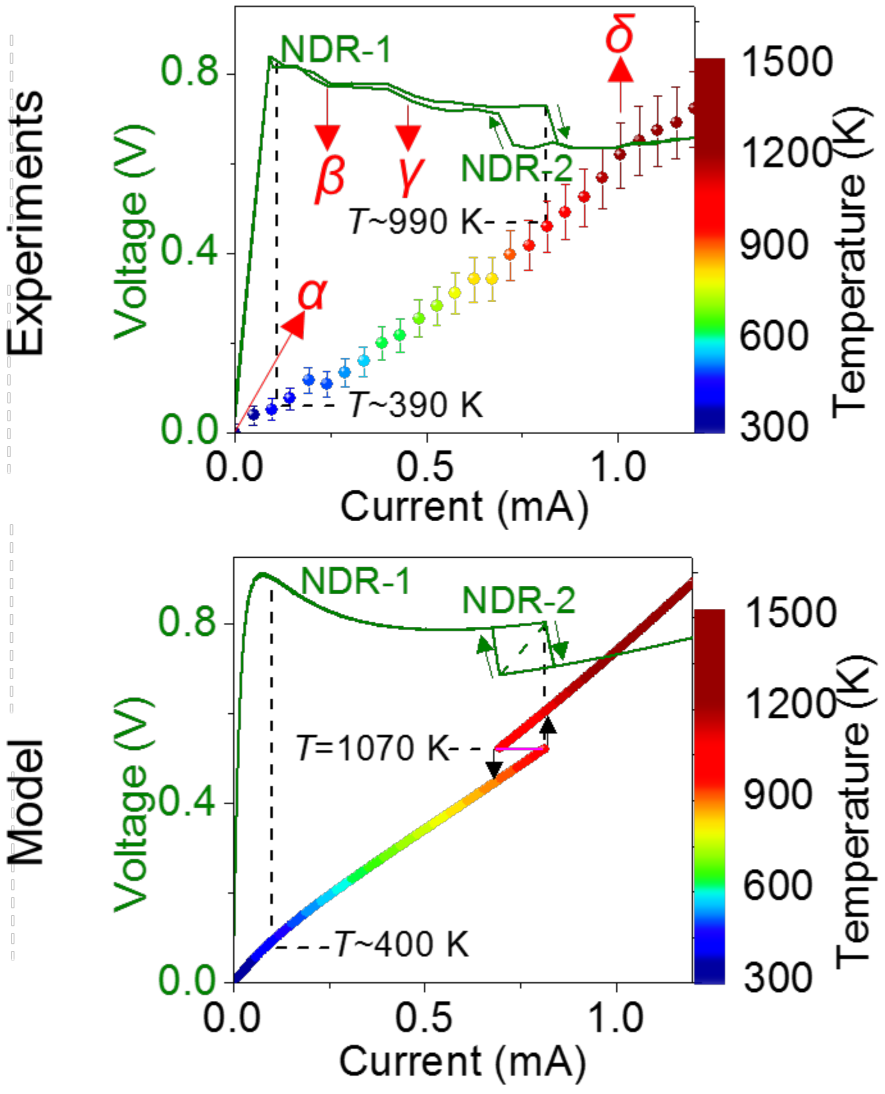
1. Smaller devices → more thermal fluctuations (< 50 nm)

2. More thermal fluctuations → higher likelihood of filament formation → failure likely

3. Dynamics become more interesting, and also noisier.

4. Most nonlinear transports allow for tunability of many of the above.

New physics in Mott insulators!



Modified 3D Poole Frenkel

$$i_m = \left[\sigma_0 e^{-0.301/2k_B T} A\left\{\left(\frac{k_B T}{\omega}\right)^2 \left(1+\left(\frac{\omega\sqrt{v_m/d}}{k_B T}-1\right)e^{\omega\sqrt{v_m/d}/k_B T}\right)+\frac{1}{2d}\right\}\right] v_m$$

$$\frac{dT}{dt} = \frac{i_m v_m}{C_{th}} - \frac{T-T_{amb}}{C_{th} R_{th}}$$

$$R_{th}(T) = \begin{cases} 1.4\times10^6 & (\text{for } T \le T_{MIT}) \\ 2\times10^6 & (\text{for } T > T_{MIT}) \end{cases}$$

Strange behavior!

Confirmed by x-ray and thermal mapping

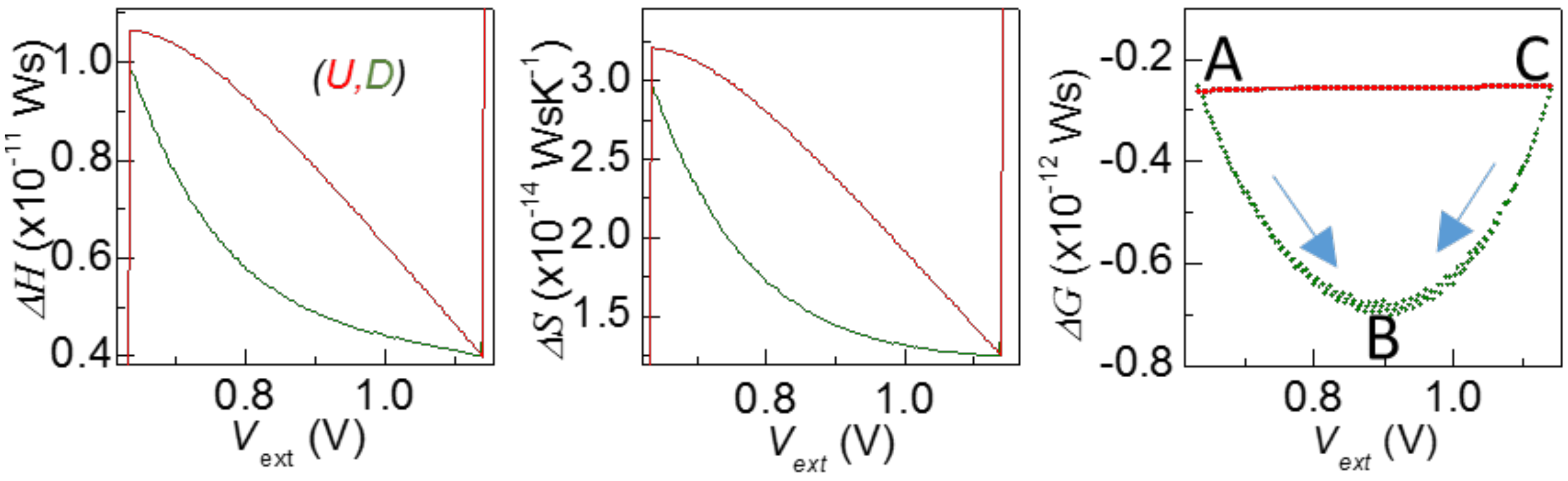
Kumar, Nature Comms. 8, 658 (2017)

Broad pointers

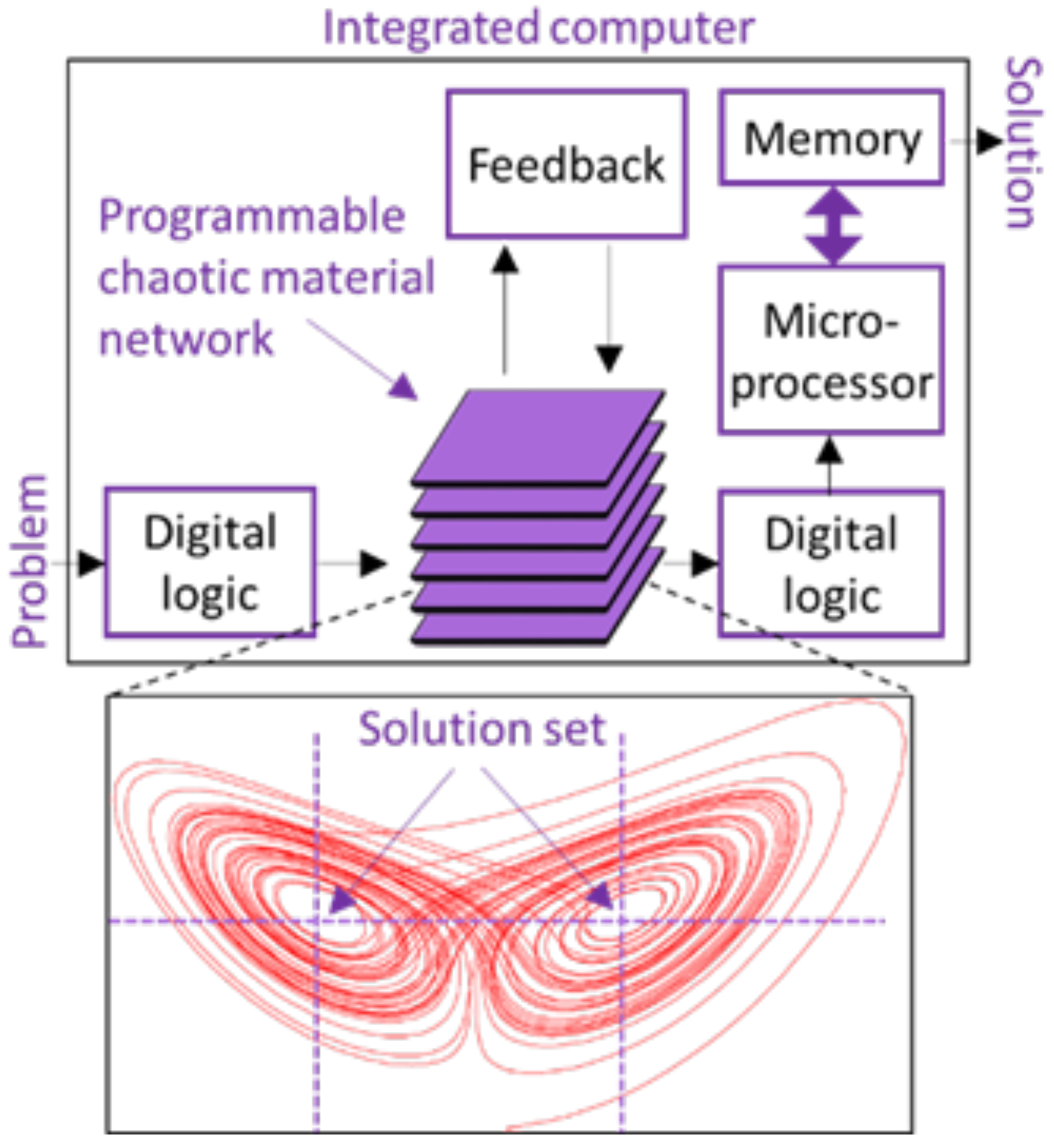
1. Practically, all future electronic devices will contain extreme nonlinearities
2. Any device model should account for
   1. Local activity
   2. Interaction with ambient state variables and their perturbations
   3. I, V, t, T – dynamics is important!
   4. Spontaneous symmetry breaking
   5. Most importantly, thermodynamic extremization

3. The search for device behaviors should be informed by simulating the performance of the architectures – this is where we do not use transistor emulators!
4. Continue to broaden the inventory of physical processes that lead to interesting device physics.

Mott transition and a "free energy well"

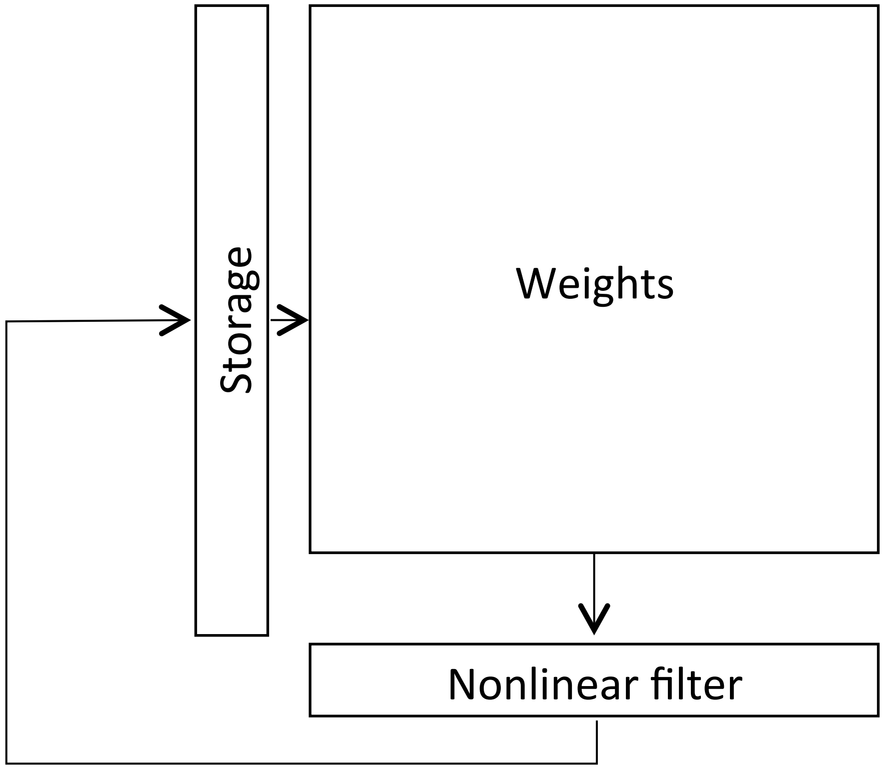


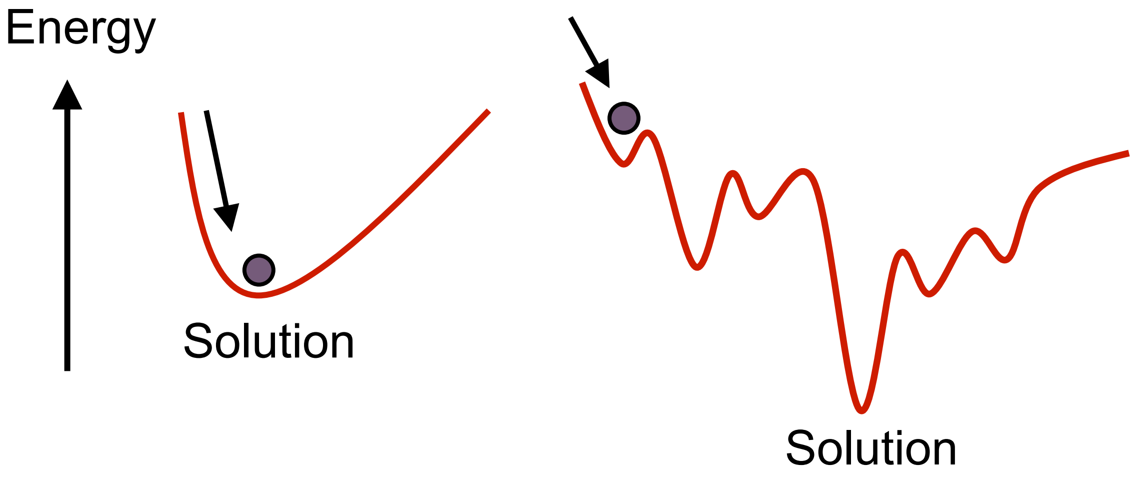
A chaos-driven computer



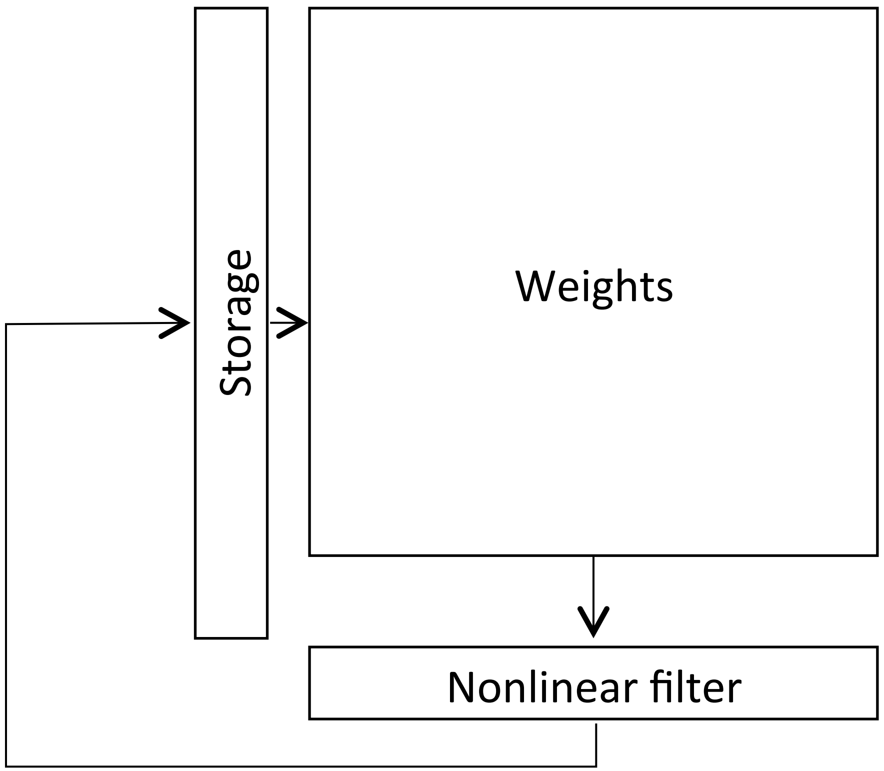
Classical analog annealing accelerators

Hopfield network





Hopfield network

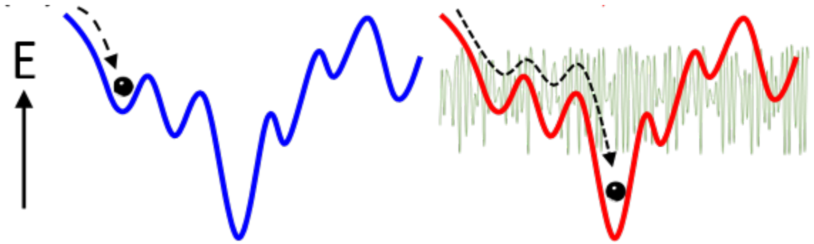


Program rule:

$$w_{(i,k),(l,j)} = -C_1 \delta_{i,l}(1-\delta_{k,j}) - C_2 \delta_{k,j}(1-\delta_{i,l}) - C_3 - C_4 D_{i,l}(\delta_{j,k+1} + \delta_{j,k-1})$$

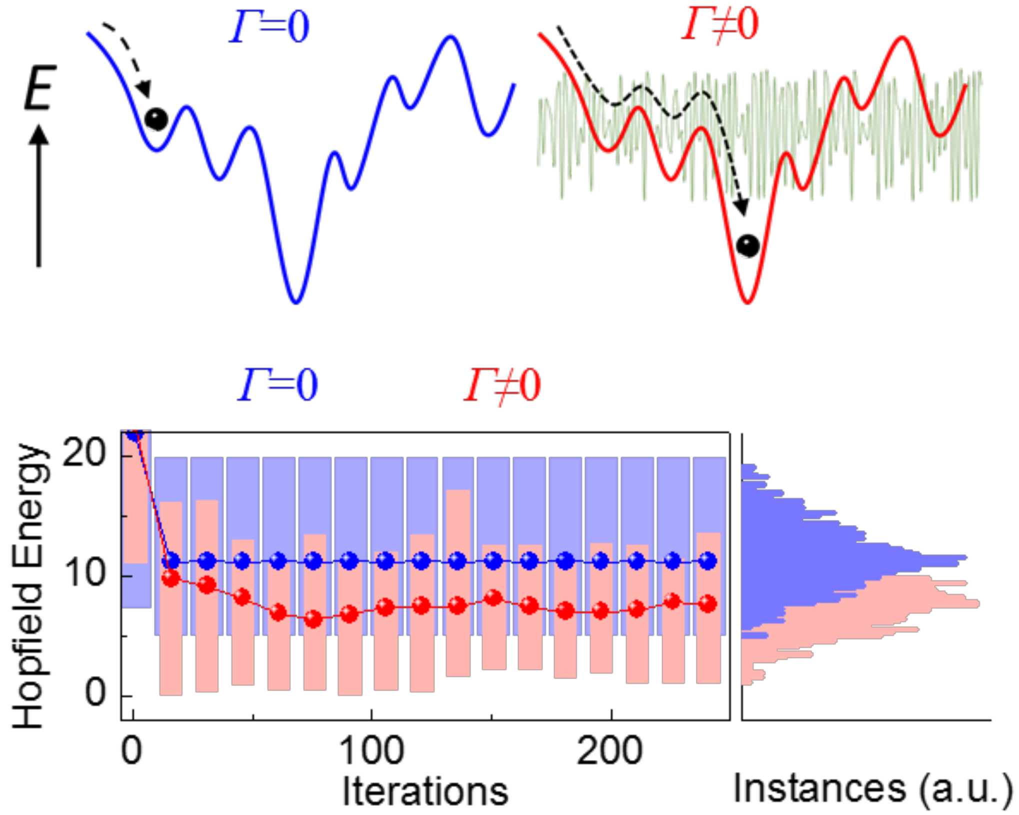
Energy function:

$$E = -1/2 \sum_i \sum_j s_{i,j} \sum_k \sum_l s_{k,l} w_{(i,j),(k,l)} + \sum_i \sum_j s_{i}$$



Kumar *et al.*, *Nature*, 548, 318 (2017)
US Patent App. 15/141,410

Statistics of many solutions with and without chaos



We only want *better* solutions quickly.

High precision → prohibitive slow downs

Literally annealing the system into its solution!

- Room temperature
- Scalable

Kumar *et al.*, *Nature*, 548, 318 (2017)

The traveling salesman problem

The Traveling
Salesman problem

Objective:

Find the shortest path

Constraints:

1. Visit every city once
2. Visit every city no more than once
3. Do not visit more than one city in a given stop

“Hard” problems

It is non-deterministic polynomial (NP) complete.

Other NP-complete/hard problems:

Gene sequencing/traveling salesman

Sudoku

Pokemon

Candy Crush

Vehicle routing

Open shop scheduling

Challenges of analogue systems

Example: graph coloring using analogue oscillators

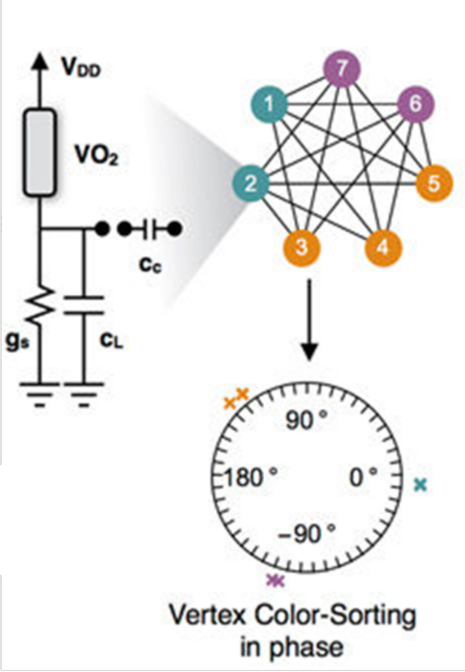
Why did analogue computers die after the 1970's?

Difficult to design
Difficult to reprogram
Did not scale
Digital emulators were error prone

Did not offer precision

Digital offered precision and scalability

In short, we used analogue systems for the wrong set of problems.



Non-scalable,
non reprogrammable

Parihar *et al., Scientific Reports,* 7, 911 (2017)

Computationally hard problems and nonlinear dynamics

"Hard" problems

↓

Nonlinear differential equations

↓

Chaotic dynamics

↓

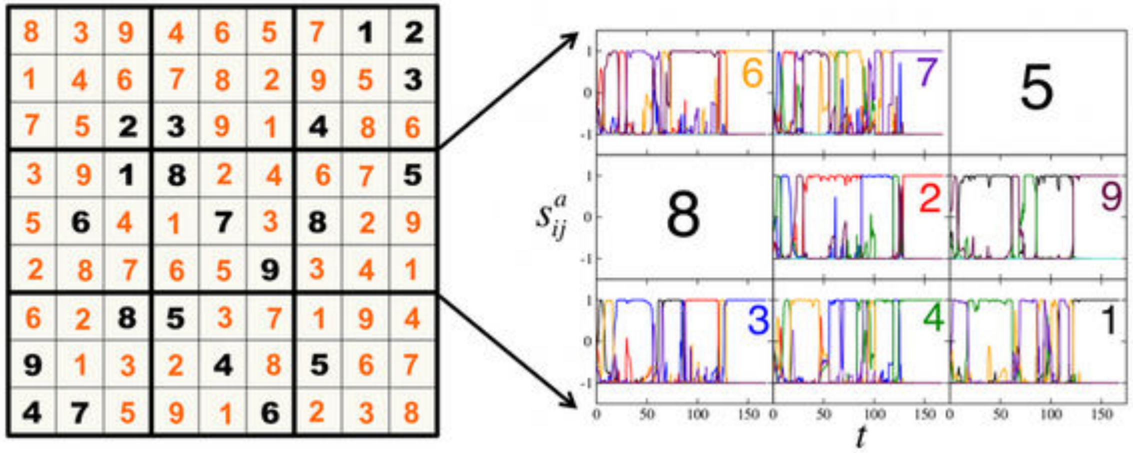
Exponential reduction in time to solution

(exponential increase in energy expenditure)

Chaos in SUDOKU

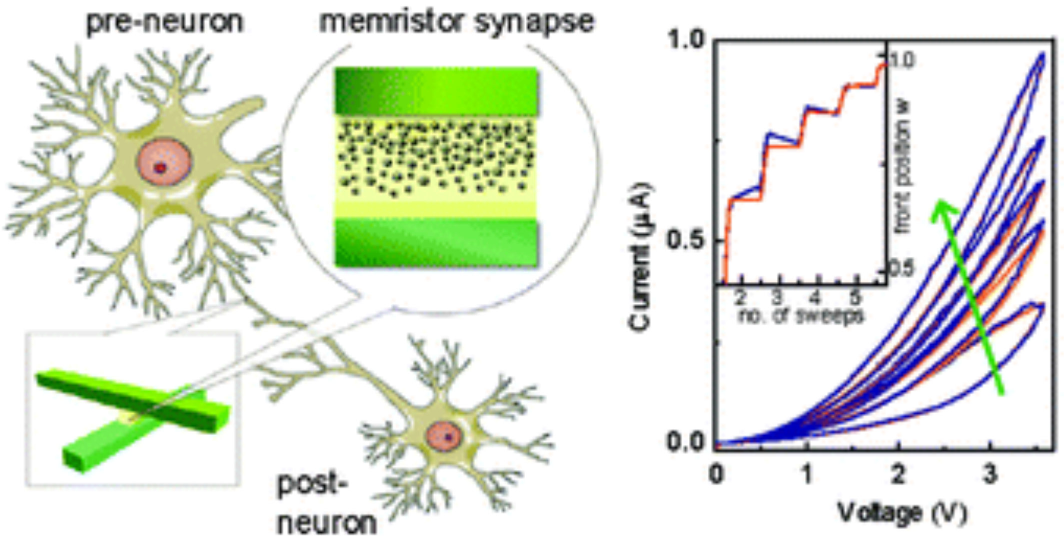
$$\frac{ds_i}{dt} = (-\nabla_s V(\boldsymbol{s}, \boldsymbol{a}))_i = \sum_{m=1}^{M} 2a_m c_{mi} K_{mi}(\boldsymbol{s}) K_m(\boldsymbol{s}) \ , \quad i = 1, \ldots, N \ , \tag{1}$$

$$\frac{da_m}{dt} = a_m K_m(\boldsymbol{s}) \ , \quad m = 1, \ldots, M \ , \tag{2}$$



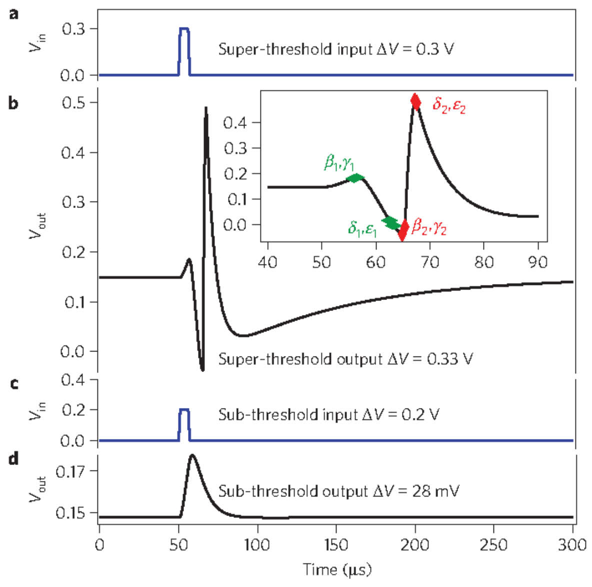
Ravasz *et al.*, *Scientific Reports*, 2, 725 (2012)

Memristors can emulate both synapses and neurons!

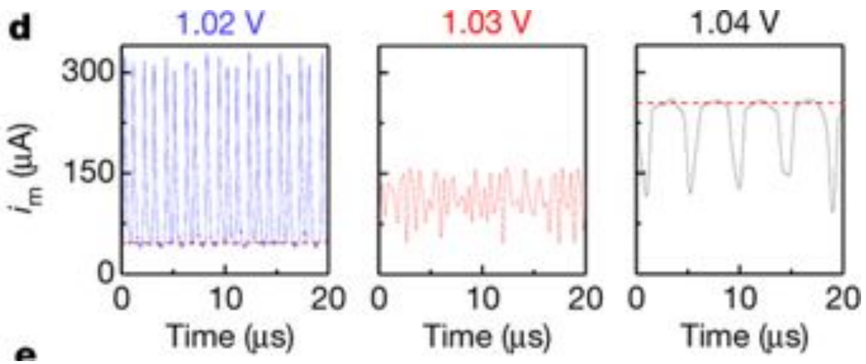


Synapse

Action potential



Edge of chaos



Pickett, Nature Materials, 2014
Kumar, Nature, 2017
Chua, “Neurons are poised near the edge of chaos”, 2012