Fundamental Thermodynamic Limits of Classical Reversible Computing via Open Quantum Systems

Karpur Shukla[1], Victor V. Albert[2,3], Michael P. Frank[4], Jimmy Xu[1]

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2: Walter Burke Inst.
3: IQIM

1: Lab for Emerging Technologies

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Computing Research

Outline

- Quantity of interest for classical reversible computing: dissipation-delay product.
  - Ingredients: GKSL with multiple asymptotic states, dissipation results for single asymptotic states, quantum speed limits.
- Representations of classical bits via decoherence free subspaces.

Motivation

- Landauer's principle already known in nonequilibrium setting, via thermal operations[1–3].
  - Average heat ejected into environment as a function of the non-unitality of the quantum channel on the system.
  - Entropy production rate for nonequilibrium (unique) steady states can be expressed in terms of the information geometry between the nonequilibrium currents[4].
- Want to characterize fundamental bounds on the dissipation of a classical reversible computational process.

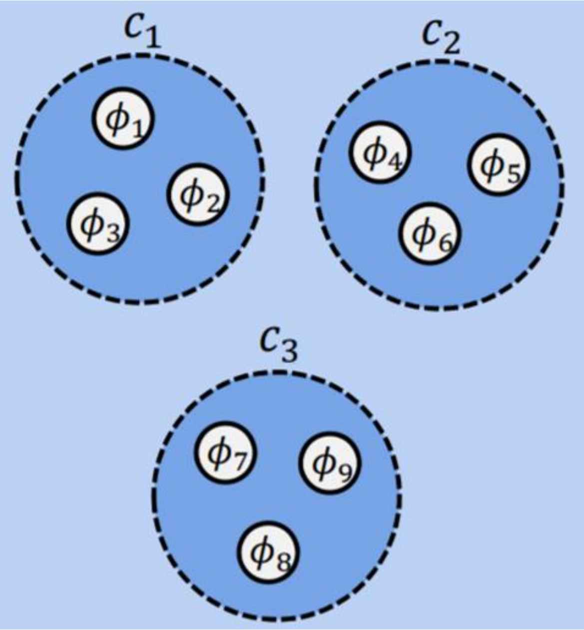
[1] – J. Goold, M. Paternostro, and K. Modi, Phys. Rev. Lett. 114, 060602 (2015).
[2] – S. Campbell *et al.*, Phys. Rev. A 96, 042109 (2017).
[3] – G. Guarnieri *et al.*, New J. Phys. 19, 103038 (2017).
[4] – G. Guarnieri *et al.*, Phys. Rev. Res. 1, 033021 (2019).

Dissipation-Delay Product

- *Power-delay product* (PDP) is a standard figure of merit in digital electronics, describing energy efficiency of logic family.
  - Product of power consumption of a logical operation and duration of that operation.
  - By analogy, want to define quantum thermodynamic bound on *dissipation-delay product* (DDP): product of dissipation of a process and time of process.
- All terms in DDP are pure quantum thermodynamics, applied to a suitable representation of classical reversible operations.
  - Likely multiple (consistent) approaches to dissipation bound: resource theory, entropy production rate.
  - Delay: time of operation from quantum speed limits.

Generalized Reversible Computing and Four Corners

- Classical reversible computing: surjective map from physical to computational states, equivalence classes.
  - All states *within* a class must have same noncomputational entropy: related by a unitary transformation.
  - We permit intra-class coherences: each class can be a decoherence-free subspace (DFS).
  - Each class in a given computational scheme must have same computational entropy: each DFS block has same dimension.
- GRC scheme can be modelled as sum of same-size DFS blocks.
  - Open quantum system approach: GKSL with multiple asymptotic states can support this.



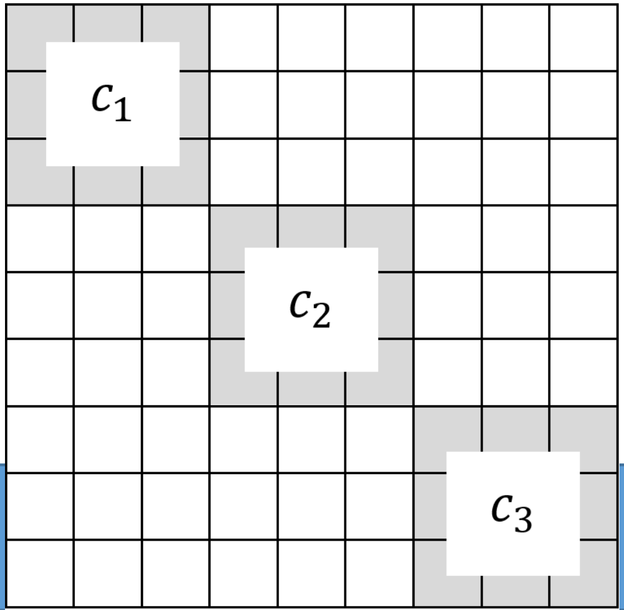
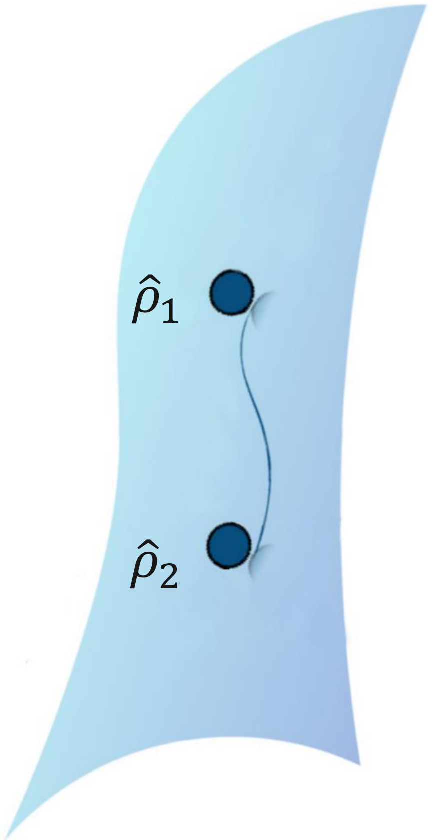


Image modified from V. Albert, arXiv:1802.00010 (Ph.D. thesis, Yale).

Thermodynamic Uncertainty Relations (TURs)

- TURs: uncertainty relation in a NESS between average currents $\langle \hat{J} \rangle$ in the system and average entropy production rate $\langle \hat{\sigma} \rangle$.
  - (Simplified) lower bound on $\langle \hat{\sigma} \rangle$ given by $\langle \hat{\sigma} \rangle \geq \langle \hat{J} \rangle^2 \cdot \left(\mathrm{Var}\langle \hat{J} \rangle\right)^{-1}$.
  - Essential for characterizing dissipation properties of autonomous machines, nanomachines, and reversible computing operations.
- TUR recently derived[4] for any system with a single NESS.
  - Dependent entirely on the information geometry of manifold of NESSs.
  - Extension to multiple asymptotic states: expect an additional dependence on the quantum geometric tensor of asymptotic space.



Metric between states given by Fisher information / Fubini-Study metric. (Single noneq. steady state.)

[4] – G. Guarnieri *et al.*, Phys. Rev. Res. 1, 033021 (2019). Image modified from here.

DDP, Bringing In Delay and Next Steps

- Asymptotic space representation: steady state-conserved current correspondence to express TUR as entropy production rate bound on computational states.
- Dissipation-delay product: dissipation (entropy production rate) and delay (quantum speed limit).
  - *Geometric* quantum speed limit[6]: quantum speed limit in terms of quantum geometry.
- Combination can give a (possibly non-tight, but still helpful) preliminary bound on DDP for classical reversible operations.
  - Related: extension of dissipation of quasistatic thermodynamic process[7] to multiple asymptotic states.

[6] – P. Poggi, Phys. Rev. A 99, 042116 (2019).
[7] – M. Scandi and M. Perarnau-Llobet, Quantum 3, 197 (2019).

Classical Computing as a Lower Dissipative Bound

- Information processing expressed as a thermal operation[5]. Dissipation:

$$\Delta E_Q \geq k_B T\left(S(\hat{\rho}_S) - S(\hat{\varrho}_S)\right) + S\left(\hat{U}_{SME}\left(\hat{\rho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E\right)\hat{U}_{SME}{}^{\dagger} \middle\| \hat{\varrho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E\right)$$

  - System S coupled to environment E and catalyst M; same as splitting E into M and E.
  - Channel: $\mathcal{E}(\hat{\rho}_S)\colon \hat{\rho}_S \mapsto \hat{\varrho}_S \coloneqq \mathrm{Tr}_M\, \mathrm{Tr}_E \left\{\hat{U}_{SME}\left(\hat{\rho}_S \otimes \hat{\rho}_M \otimes \hat{\rho}_E\right)\hat{U}_{SME}{}^{\dagger}\right\}$.
  - First term: information cost of classical IP. Second term: quantum IP.

- Classical IP is a *lower* dissipative bound! Quantum IP can be equal at best.
  - Classical IP: signal states correspond to orthogonal quantum states.
  - Pure unitaries and single input & output operations match classical IP dissipation bound.

[5] – D. Bedingham and O. Maroney, New J. Phys. 18, 113050 (2016).