CREU 2016-2017 Final Report: Performing meta-analysis of dimensionality reduction techniques to create a dataset and method taxonomy

CREU Team Mentor Dr. Shirley Yap, CSU East Bay
CREU Team Member Linda Beverly, CSU East Bay
CREU Team Member Perry Aliado, CSU East Bay

I Goals and Purpose

Goal: Performing meta-analysis of dimensionality reduction techniques to create a dataset and method taxonomy.

Purpose: Geometric data analysis combines theoretical mathematics with applications to real-world practical problems. The first part of our project is learning the relevant geometries of these problems, which will include the study of manifolds and differential geometry. We will then study the underlying geometries of ISOMAP, Locally Linear Embedding (LLE), and Local Tangent Space Alignment (LTSA) dimension reduction techniques. After understanding these theoretical bases, we will apply the algorithms to real-world data. After testing various data sets with the various algorithms, we will compare the efficacy of the algorithms. The end goal of our project will be to complete a meta-analysis of the different techniques and to produce a taxonomy of the kinds of datasets that can be best modeled using these different dimensionality reduction techniques.

We studied Locally Linear Embedding (LLE), PCA, and ISOMAP. We applied LLE to a new and unique application utilizing LLE on video data. We found that this began leading us into neural networks and deep learning for image recognition.

We applied Locally Linear Embedding, a method of geometric dimensionality reduction, to identifying similar color regions over time within video. This technique preserves the underlying geometric information. Identifying similar regions of color may be used to preprocess information that may be used to prepare the information for edge detection, video compression and psychovisual optimization.

We wanted to process raw video information via Locally Linear Embedding (LLE) to extract neighborhood preserving distances. We wanted to show how location, time, and color can be used to cluster related information within the video source. This work is part of a larger project in geometric data analysis. The goal is to complete a meta-analysis of different techniques, and to produce a taxonomy of the kinds of datasets that can be best modeled using different dimensionality reduction methods.

II Related Work

Locally Linear Embedding, introduced by Saul & Roweis [2], is an unsupervised learning algorithm that is locally linear and preserves geometric and topological information [3]. LLE computes lower dimensional embeddings of higher dimensional inputs while preserving relative neighborhood distances.

The LLE algorithm consists of three logical steps [1]:

1. Find neighbors; in our work, we chose to use k_{th} nearest using Euclidean distances.
2. Solve for reconstruction weights, which involves finding the Gaussian distribution of the distances of every point to each of its neighbors, and representing each neighbor's influence toward the barycenter point as a fraction of total influence which must sum to 1. Populate a $p \times p$ weight matrix W such that all non-neighbor points are zero, and all neighbor points are the reconstruction weights just found. Use the weights matrix W to compute matrix M as $(I - W)^T \cdot (I - W)$.
3. Compute embedding coordinates matrix Q by finding the bottom d+1 eigenvectors of M corresponding to its smallest d+1 eigenvalues; discard the eigenvector corresponding to eigenvalue 0.

References

[1] Roweis, Sam T., and Lawrence K. Saul Lleintro.pdf. Accessed May 2, 2016. `https://www.cs.nyu.edu/~roweis/lle/papers/lleintro.pdf.`

[2] Roweis, Sam T., and Lawrence K. Saul *Nonlinear Dimensionality Reduction by Locally Linear Embedding.* Science 290, no. 5500 (December 22, 2000): 232326. doi:10.1126/science.290.5500.2323.

[3] Ziegelmeier, Lori, Michael Kirby, and Chris Peterson. *Locally Linear Embedding Clustering Algorithm for Natural Imagery.* arXiv:1202.4387 [Cs, Math], February 20, 2012. `http://arxiv.org/abs/1202.4387.`

[4] Tenenbaum, Joshua, Vin de Silva, and John Langford. *A Global Geometric Framework for Nonlinear Dimensionality Reduction. 2000, Science.* 290, no. 5500: 2319-2323.

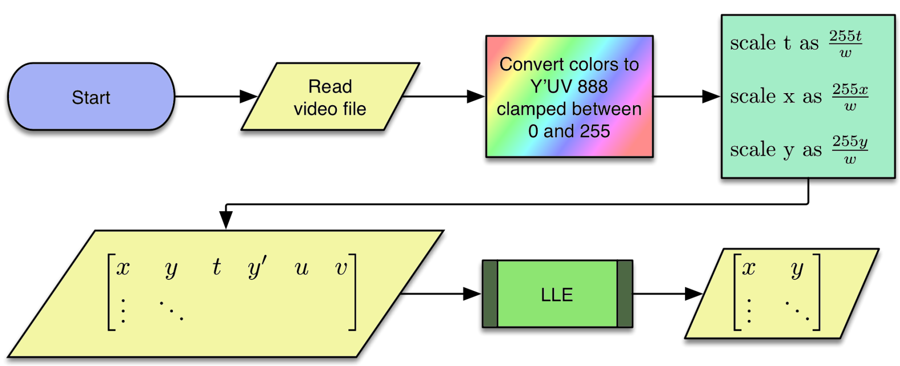
[5] Saul, Lawrence K., Sam T. Roweis, and Yoram Singer. 2004. *"Think Globally, Fit Locally: Unsupervised Learning of Low Dimensional Manifolds."* Journal Of Machine Learning Research 4, no. 2: 119-155. Academic Search Complete, EBSCO host (accessed May 5, 2016).

III Process

We began with the scholarly article "An Introduction to Locally Linear Embedding." It took several months to understand the details of this paper. Along the way we studied Principal Component Analysis and ISOMAP. We also studied differential geometry. The data analysis algorithms and methods cover many different areas of mathematics including but not limited to linear algebra, statistics, graph theory, and real analysis.

Our approach was to study different dimensionality reduction algorithms in depth to understand how and why they work so that we could be able to use that information to help create a taxonomy of relevant dataset methods to different types of data. We focused on Locally Linear Embedding and an application to video data in order to see if LLE was useful for clustering like regions over space and time. We used the scikit-learn procedural version of Locally Linear Embedding to test on video source data.

For the video application: To begin, we read in video source data and converted the color information to Y'UV, a planar color space that has been scaled to emphasize human perceptible color differences. Feature scaling was then applied to normalize the data and eliminate feature bias. Each pixel was then represented as a six dimensional tuple (X, Y, Time, Y', U, V), creating a $P \times 6$ matrix.



Next, we performed LLE on the data with the parameters of k = 16, 32, and 64 neighbors and an embedding dimensionality of 2.
Finally we analyzed the embedding and identified the clusterings found in each neighborhood choice. To determine the efficacy of the clustering, a simple Gaussian KDE was performed over each sample region.

IV Results and Discussion

Video of 3d Animation

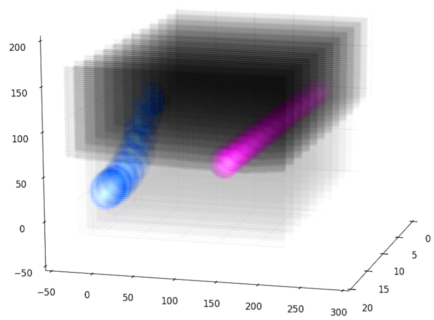


Figure 1: Transparent pixel representation of 13 frames of video source.

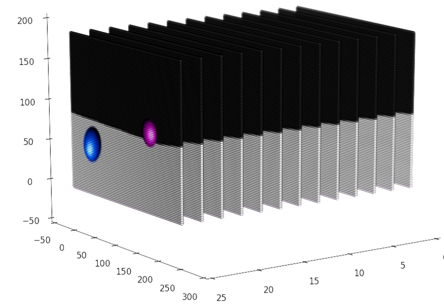


Figure 2: Opaque pixel representation of 13 frames of video source.

Data proximity is preserved in the lower dimensional embedding, that is, points that have nearness within a local neighborhood in the higher dimension maintain their nearness in the embedding. Because of this property, points that are nearer in all dimensions tend to cluster together in the lower dimensional embedding. Figures 3 and 4 represent these tight clusterings within the 2D embedding of the 3D animation video.

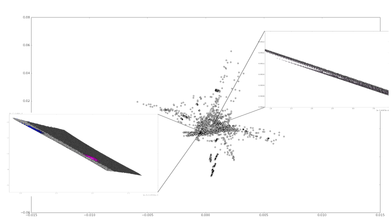


Figure 3: The 2D embedding and two extremely zoomed views of two interesting areas.

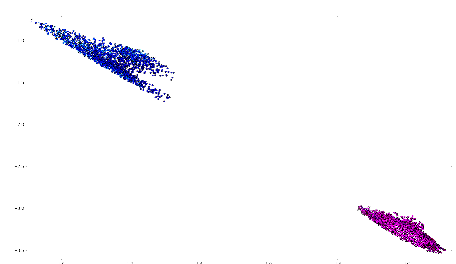


Figure 4: The isolated blue and purple ball through time from the bottom-left zoomed region of Figure 3.

We then compared the results of this process on video of natural imagery to determine if the process scales to more complex input. Our second dataset comes from five frames of natural video of a butterfly in motion and plant life.

Our results seem to indicate that that the process can be applied to video sources of varied types and yield similar results.

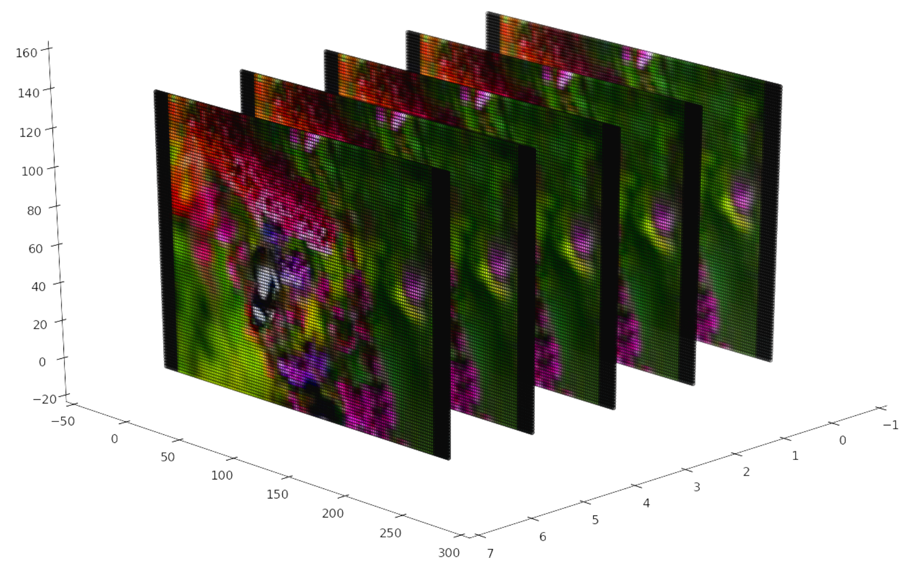


Figure 5: 5 Frames of video of natural imagery.

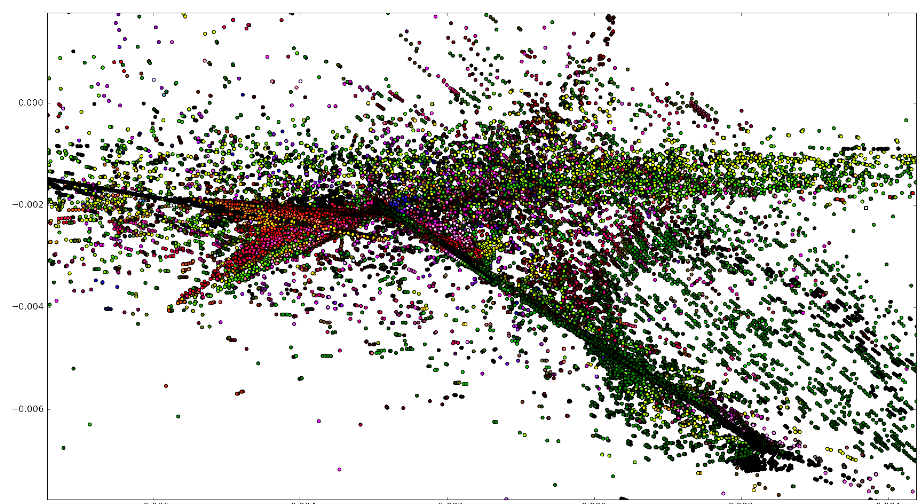


Figure 6: Embedding of natural video into 2d.

LLE was able to identify and cluster similar-color regions over temporal and spatial domains as is visually demonstrated in the figures of the embeddings of both the 3D animation and the video of natural imagery above.

We were able to find chromatic clusters within the video samples. These clusters can now be used to identify active regions within the video and track similar regions through time. With these regions identified, we can make bitrate allocation decisions, and may be able to improve psychovisual enhancement techniques in video compression applications.

As a comparison, the Swiss Roll is a standard example for demonstrating how LLE preserves distance information. The colors are clustered based on distance from the origin and their distance ratios are preserved through the embedding.

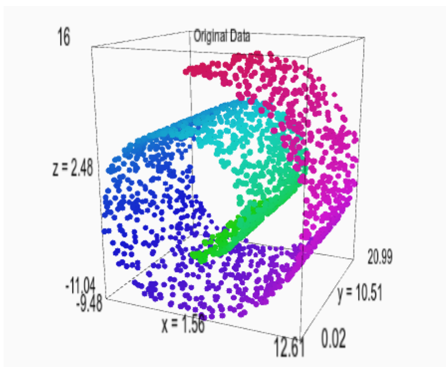


Figure 7: Randomly generated 3d Swiss Roll.

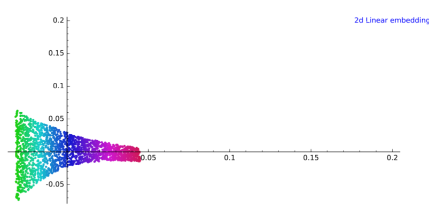


Figure 8: Embedding of the Swiss Roll into 2d.

V Future Work

- Neural networks for image recognition, focusing on digit and handwriting with Convolution Neural Networks.
- Preprocess data with LLE and other dimensionality reduction methods and combine with neural networks and deep learning.
- Create iterative kd-tree for populating nearest neighbors in less memory, sacrifice time for space.
- Create an open-source iterative version of Locally Linear Embedding that can be used to process larger sets of higher dimensional data in less memory.
- Optimize and parallelize entire pipeline taking advantage of details known about input data.

VI Web Links

`http://blog.hyperspacedonuts.com`

VII Presentations and Publications

- Locally Linear Embedding of Chromatic Clusterings in Temporal and Spatial Domains Undergraduate Poster Session, January 6, 2017, JMM 2017, Atlanta, Georgia
- Locally Linear Embedding of Chromatic Clusterings in Temporal and Spatial Domains Poster Session, MAA Golden Section, March 4, 2017, Santa Clara, California
- Locally Linear Embedding of Chromatic Clusterings in Temporal and Spatial Domains Talk, NCUMC, March 25, 2017, Sonoma State University, Rohnert Park, California

Abstract submitted for future conference:

- Locally Linear Embedding of Chromatic Clusterings in Temporal and Spatial Domains Talk, MathFest, July 26 - 29, 2017, Chicago, Illinois