# Optimizing in a Strategic World: An Invitation to Algorithmic Game Theory

Anna R. Karlin
University of Washington

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# Speaker & Moderator



Anna Karlin

Anna R. Karlin, is the Microsoft Professor of Computer Science and Engineering at the Paul G. Allen School of Computer Science at the University of Washington. She received her Ph.D. from Stanford University in 1987. Before coming to the University of Washington, she spent 5 years as a researcher at (what was then) Digital Equipment Corporation's Systems Research Center. Her research is primarily in theoretical computer science: the design and analysis of algorithms, particularly algorithmic game theory, probabilistic and online algorithms. She also works at the interface between theory and other areas, such as economics and game theory, data mining, operating systems, networks, and distributed systems. In addition to her many papers, she is coauthor of the book "Game Theory, Alive" with Yuval Peres, published in 2017 by the American Mathematical Society. She is a Fellow of the Association for Computing Machinery and a member of the American Academy of Arts and Sciences.



Sheila Castañeda

Sheila Castañeda is Professor Emerita in the Computer Science Department at Clarke University in Dubuque, Iowa, where she taught from 1980-2014. Her main areas of interest and teaching responsibility are in database systems and programming languages.

She received her master of science degree in computer science from the College of Wisconsin – Madison in 1979. She is also a member of the Association for Computing Machinery (ACM) and its Special Interest Group on Computer Science Education (SIGCSE). Away from the office, she enjoys gardening, baking, traveling and spending time with her family.



# Anna Karlin University of Washington

algorithm design and analysis algorithmic game theory



Sophie (19)
Gabi (19)
Evan (old, like me)





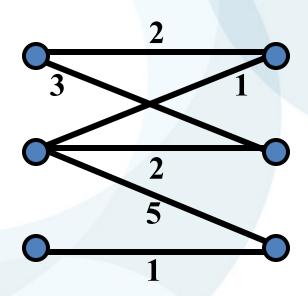
# An Example

### **Classical Optimization Problem:**

#### **Maximum Weighted Matching**

**Input:** Weighted Bipartite Graph

Output: Matching that maximizes the sum of matched edge weights.





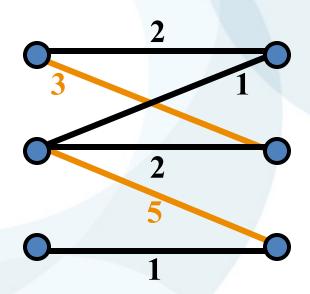
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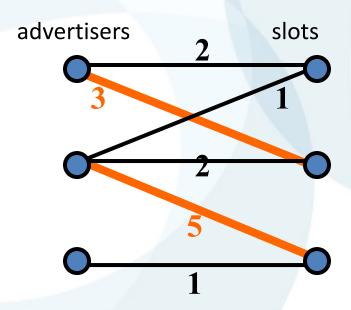


# **Example Application**

#### **Selling advertising slots**

A search engine has advertising slots for sale

Advertisers are willing to pay different amounts to have their ad shown in a particular slot



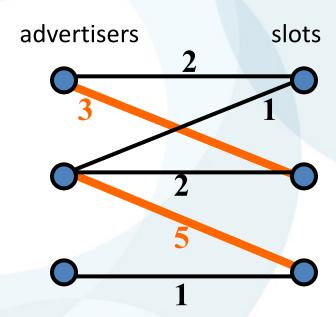
**Optimal Search Engine Revenue = maximum weighted matching** 



## **Private Values**

Algorithm must solicit values

Advertisers may lie to get a better deal

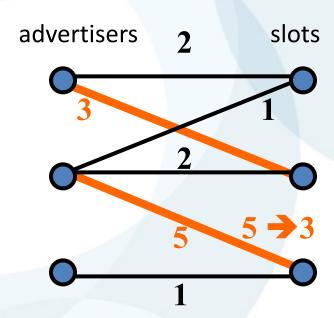




## **Private Values**

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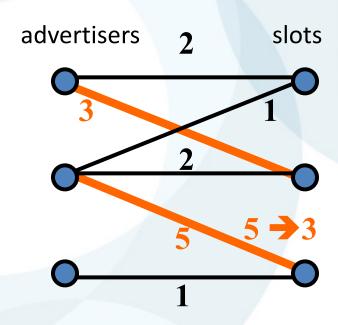




## **Private Values**

Algorithm must solicit values

Advertisers may lie to get a better deal



What if all advertisers speculate?



# Big Picture

Many problems where input is private data of agents who will act selfishly to promote best interests

Resource allocation
Routing and congestion control
Electronic commerce

#### **Fundamental Question:**

How do we optimize in a strategic world?

Use ideas from game theory and economics.



## Outline of talk

Brief background in game theory.

Brief survey of two of the major areas in algorithmic game theory:

Price of anarchy using example of selfish routing

Mechanism design = "incentive engineering"



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Game: players, strategies, payoff functions

Given a set of players, each with a set of strategies, each motivated by self-interest, what constitutes rational behavior?

	Swerve	Straight
Swerve	(0,0)	(-1,+1)
Straight	(+1,1)	(-5,-5)



## Poll 1

If you're playing Chicken against me (say I'm the row player and you're the column player) and you knew I was going to swerve, what would you do?

- A. Swerve
- B. Straight

	Swerve	Straight
Swerve	(0,0)	(-1,+1)
Straight	(+1,1)	(-5,-5)

## Poll 2

If you're playing Chicken against me (say I'm the row player and you're the column player) and you knew I was going to go straight, what would you do?

- A. Swerve
- B. Straight

	Swerve	Straight
Swerve	(0,0)	(-1,+1)
Straight	(+1,1)	(-5,-5)

Fundamental Concepts: various equilibria

**Dominant strategies**: each player has a well-defined best strategy no matter what any of the other players do.

	Swerve	Straight
Swerve	(0,0)	(-1,+1)
Straight	(+1,1)	(-5,-5)

## Poll 3

Does the game of Chicken have dominant strategies, i.e. do you have a single strategy that you use no matter what I do?

A. Yes

B. No

	Swerve	Straight
Swerve	(0,0)	(-1,+1)
Straight	(+1,1)	(-5,-5)

Fundamental Concepts: various equilibria

**Dominant strategies**: each player has a well-defined best strategy no matter what any of the other players do.

Nash equilibria: each player's strategy is best response to opponents' strategies.

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#### Chicken

	Swerve	Straight
Swerve	(0,0)	(-1,+1)
Straight	(+1,-1)	(-5,-5)

	Heads	Tails
Heads	(+1,-1)	(-1,+1)
Tails	(-1,+1)	(+1,-1)

## Poll 4

Does the game of Matching Pennies have a Nash equilibrium (a square in which each player's strategy is a best response to the strategy of the other player)?

- A. Yes
- B. No

	Heads	Tails
Heads	(+1,-1)	(-1,+1)
Tails	(-1,+1)	(+1,-1)

Fundamental Concepts: various equilibria

**Dominant strategies:** each player has a well-defined best strategy no matter what any of the other players do.

Nash equilibria: each player's strategy is best response to opponents' strategies.

•		1/2	1/2
		Heads	Tails
1/2	Heads	(+1,-1)	(-1,+1)
1/2	Tails	(-1,+1)	(+1,-1)

Fundamental Concepts: various equilibria

**Dominant strategies:** each player has a well-defined best strategy no matter what any of the other players do.

Nash equilibria: each player's strategy is best response to opponents' strategies.

## **Matching Pennies**

Yes, if we allow **mixed** (or randomized) strategies

		1/2	1/2
		Heads	Tails
/2	Heads	(+1,-1)	(-1,+1)
/2	Tails	(-1,+1)	(+1,-1)

John von Neumann: Every zero-sum game has a mixed Nash equilibrium.

John Nash: Every finite game has a mixed Nash equilibrium.

#### Chicken

	Swerve	Straight
Swerve	(0,0)	(-1,+1)
Straight	(+1,-1)	(-5,-5)

١.	1/2		1/2
		Heads	Tails
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Brief background in game theory.

Brief survey of two of the major areas in algorithmic game theory:



Price of anarchy using example of selfish routing

Mechanism design = "incentive engineering"



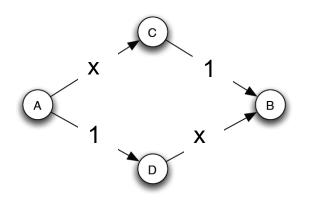
## Price of Anarchy

- Explores games that arise "in the wild", such as in Internet settings.
- Tries to understand the impact of selfish behavior on society by comparing the overall performance attained in equilibrium when players behave selfishly to the performance that could be attained if decisions were made by a centralized authority.



## Selfish Routing [Roughgarden, Tardos]

- Model network as directed graph.
- We assume network users are selfish -- in equilibrium each user will choose a route that minimizes their travel time, given what everyone else is doing.
- In this example, equal balance of traffic is a Nash equilibrium.



Average latency = 3/2

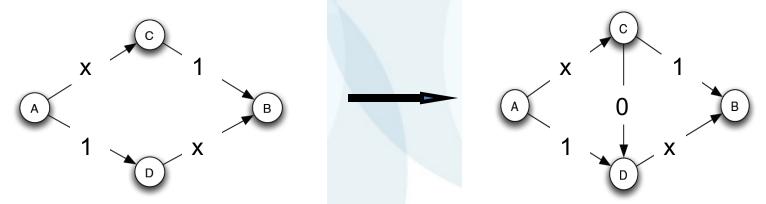


Edge labels: latency as function of fraction of traffic

## Braess's Paradox

- Small changes can lead to counterintuitive behavior.
- Example: Government builds a new, very fast highway.

Edge labels: latency as function of fraction of traffic



Average Latency

in Nash Eq = 3/2



## Poll 5

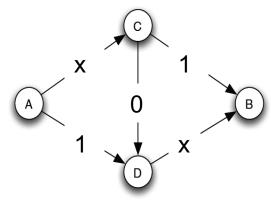
What is the expected delay in the selfish equilibrium?

A. 3/2

B. 2

C. 3

D. 4



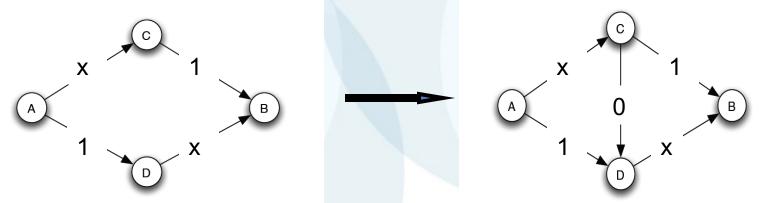
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Average Latency

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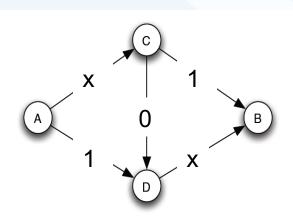
Average Latency in Nash Eq = 2

Unique Nash Equilibrium in new network leads to worse travel time for everyone!



# Price of Anarchy [Koutsoupias, Papadimitriou]

Example of larger phenomenon: selfishness can be bad for society. How bad?



Selfish equilibrium: 2

Social optimum: 3/2



# Price of Anarchy

How bad can selfishness be for society?

Previous example is **worst case** (any network, affine latency functions) [Roughgarden, Tardos]

Price of Anarchy = Avg latency at selfish equilibrium Avg latency at social optimum

is always at most 4/3

Ongoing work: other games and equilibria



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Mechanism design = "incentive engineering"



# Mechanism Design

Given desired goals (e.g. maximizing profit or global happiness), design the game in such a clever way that rational players, motivated solely by self interest, will end up achieving the designer's goals!

## Examples:

- Designing a system of tolls on roads (pricing link usage in computer networks).
- Selling advertisement slots on a search engine.



## Killer Application



ad auctions

Search

Advanced Search

Show options...

Results 1 - 10 of about 29,000,000 for ad auctions. (1.44e-15 epoc

#### oduction to the Google Ad Auction



9 min - Mar 11, 2009 - ★★★★

Our Chief Economist, Hal Varian, explains the AdWords **Ad Auction** and how your max CPC bid and quality score determine how much you pay for a ...

www.youtube.com/watch?v=K7I0a2PVhPQ - Related videos

#### mpeting Ad Auctions: Multi-homing and Participation Costs — HBS ...

22, 2010 ... Joining ad platforms can attract substantial regulatory attention: In rember 2008, the Department of Justice planned to file antitrust ... wk.hbs.edu/item/6354.html - Cached

#### nt Advertising in Print Media - Newspaper Advertising ...

it advertising, newspaper advertising, magazine advertising and per-inquiry print ertising made easy. ... RFP/ Print **Advertising Auctions** ... w.mediabids.com/ - Cached - Similar

#### slice of pizza: Ad Auctions Workshop 2010

6th **Ad Auctions** Workshop will take place on June 8th, collocated with the EC ference. The call for participation is now available. Deadline April 14. ... sliceofpizza.blogspot.com/2010/.../ad-auctions-workshop-2010.html - Cached

#### ws results for ad auctions



Orange, OpenX Set Out To Build Pan-European Ad Exchange Ecosystem - 1 day ago

Sponsored Links

#### Create An Ad Here

Target New Customers With Ea Start Making Every Click Coun www.Google.com/AdWords

#### Print Ads

Newspaper & magazine ads ha never been easier to buy. www.mediabids.com

#### Ad Auctions

Looking for **Ad Auctions**? Find exactly what you want tod Yahoo.com

See your ad here »



# Selling a single item

Goal: design auction to maximize profit

Run a sealed-bid auction. Each bidder submits a bid.

Most natural allocation and pricing:
Highest bidder wins and pays what they bid.

Is this (first-price auction) a good choice?



## First-price auction

What will happen?

To answer need to understand payoff structure.

#### Standard bidder model:

Each bidder has a private value v<sub>i</sub> for the item.

Player utility: [v<sub>i</sub> - price] if wins, 0 otherwise

Each player submits a bid b<sub>i</sub> for the item

#### Problems with first-price auction

Players will "shade" their bids.

But they have to speculate about values/bids of others => figuring out how to bid is complicated.

Outcome unpredictable.



## The Vickrey 2nd price auction.

### Alternative auction design:

Players submit bids.

The player that submitted the highest bid wins, but only pays the second highest bid submitted.



## Poll 7

#### Vickrey 2<sup>nd</sup> price auction

Players submit bids.

The player that submitted the highest bid wins, but only pays the second highest bid submitted.

Suppose the players values are (68, 100, 2) and you are the player with value 100.

What is your best bid?

- A. 100
- B. 69
- C. Anything above 68.
- D. 50



# The Vickrey 2nd price auction.

### Alternative auction design:

Players submit bids.

The player that submitted the highest bid wins, but only pays the second highest bid submitted.

(68,100, 2) player submitting 100 wins at price 68

Key property: it is in each player's best interest to bid their true value, no matter what the other players do!



## Vickrey 2nd Price Auction

Key property: it is in each player's best interest to bid their true value, no matter what!

#### Remarks:

Players don't need to think about how to bid, because it is in their best interest to tell the truth!

The item will go to the player that values it the most!

It has been proven\* that, in equilibrium, seller profit using 1st and 2nd price auction is the same.

\* under modest assumptions



## Mechanisms for profit maximization

#### Two strands of research:

#### Bayesian:

- Agents values assumed to come from publicly known prior distributions.
- Goal: to do well in expectation

#### Prior-free

- What if we don't want to assume a prior?
- Want to do well in worst case



### Issues with Bayesian Profit Maximization

Not necessarily realistic to assume priors known Not necessarily robust to small deviations.

#### Key questions:

How do we design truthful mechanisms for profit maximization that work well without priors?

How do we evaluate these mechanisms?

Approach: competitive analysis framework



## Example: Digital Goods Auction

#### Given

Unlimited number of copies of identical items for sale

n bidders, bidder i has private value  $v_i$  for obtaining one item (and no additional value for more than one)

Goal: Design truthful auction to maximize profit



## Maximizing Profit: A Competitive Analysis Framework

Goal: truthful profit maximizing basic auction

There is no auction that is best on every input. How then should we evaluate auctions?

Competitive analysis

Compare auction profit to "profit benchmark" OPT(v).

#### **Definition:**

An auction is c-competitive if for all v its profit is at least OPT(v)/c

## Profit Benchmark for Digital Goods Auction

Define OPT(v) = optimal fixed price revenue

Example: v = (3, 2, 2, 1, 1) OPT(v)= 6

#### Cannot implement this truthfully!

How do we design competitive truthful auction?

Key observation: for an auction to be truthful has to be bid independent – price offered to bidder is independent of that player's bid.

Most natural choice for f: Optimal fixed price for b<sub>-i</sub>

Doesn't work!

Generic Truthful Auction:  $\mathrm{BI}_f$ 

On input  $\mathbf{b}$ , for each bidder i:

1. 
$$p \leftarrow f(\mathbf{b}_{-i})$$
.

- 2. If  $p \leq b_i$ , sell to bidder i at price p.
- 3. Otherwise, reject bidder i.

## Digital Goods Auction

no deterministic symmetric auction achieves any constant competitive ratio.

Must turn to randomized auctions.

Key idea: random sampling



## Random Sampling Auction for Digital Goods

- Randomly partition bids into S and S'
- Offer optimal price for S' to bidders in S
- Offer optimal price for S to bidders in S'

#### Theorem [GHKSW]

This auction is truthful and achieves a constant fraction of the optimal profit in expectation.



### **Conclusions**

Algorithmic game theory is an exciting and vibrant research area.

Computer scientists have brought quite a bit to the table.

New problems or new light on old problems.

Different/new techniques (e.g. randomization, approximation and reductions)

Computational complexity

Communication complexity and distributed implementation Different performance measures, such as worst-case.

Thank you for your attention!!



### **Conclusions**

If you'd like to know more, check out By Nisan, Roughgarden, Tardos, Vazirani

And if you want to know more about game theory, check out my new book!



# **Game Theory, Alive**with Yuval Peres

Published in 2017 by the American Mathematical Society
Available online for free on my web page:
http://homes.cs.washington.edu/~karlin/GameTheoryBook.pdf

# What does a good grad school application look like?

## Poll 7

Are you interested in applying to a Masters or Ph.D. program?

- A. Yes, definitely, Ph.D.
- B. Yes, definitely, Masters.
- C. I'm not sure yet.
- D. No.



# Why consider graduate school?

- You want to solve big and important problems
- You love to be creative and want a lot of independence and control over the choice of problems you address
- You want to make important and long-lasting contributions to the field
- You would enjoy being an expert on a particular area in computer science
- Starting salaries for Bachelor's degrees are high; starting salaries for M.S. and Ph.D. are often higher
- Your chosen career (e.g., professor or research scientist) requires it



# Focus today

Applying to a Ph.D. program only makes sense if....

- you know you want to do research and possibly teaching.
- you have done some research.

If one of the above doesn't hold, consider applying to an masters program.

# So you've decided to apply to grad school....

### Components of an application:

- Basic contact info
- transcripts
- letters of recommendation (usually 3)
- statement of purpose (goals/research/intent)
- resume
- test scores (GRE, TOEFL / IELTS)
- fee



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## **Grades**

Best to have a GPA in CS, Math and Engineering that is 3.5 out of 4.0 or better.

But....

A GPA of 4.0 with no research experience will not get you into a really good Ph.D. program.

Consider taking a graduate course or two while you are an undergrad!

# Statement of purpose

Admissions committee is looking for a "research statement":

- General areas of research that interest you and why.
- 2. Describe research problems you have worked on. What, why, how.
- 3. Explain why you want a Ph.D.
- 4. Explain why you want to go to this particular university.



# Implication: should try to do research!

How to get a research experience?

- Work with a professor at your undergraduate institution
- Apply for summer internship. Google "summer internships for undergraduates" and "REU computer science".
- Do a Masters degree/project



# Why do research?

- It will help you figure out if you actually want to go to grad school!
- It will give you something to talk about in your statement of purpose.
- You will get to know a professor who can write you a letter of recommendation.



# How to ask a professor to do research with him/her?

- Make a list of suitable professors (check out web pages)
- Prepare a summary sheet about yourself.
- Talk with your undergraduate advisor to find out about research opportunities.
- Pick a professor and go to his/her office hours (or if that is not possible) schedule appointment.

# Be prepared to answer

- Do you have ideas about what kind of research you're interested in?
- Which of my projects are you most interested in?
- Describe your programming background.
- Describe your math background.



# Most will say no. Don't give up!

- Try a grad class with a professor whose research interests you.
- Get to know some grad students and see if you can help them out with something.
- Find out about seminars/group meetings and ask if you can attend.



## When to start research?

- Ideally, beginning of your junior year.
- Could be focus of summer between junior and senior year.



# Most important part of application: letters of recommendation

Probably will need 3 letters.

 "Did well in class" is not nearly as good a letter as one that can speak to your self-motivation, research potential, initiative, independence and drive!



# Asking for recommendations

"Do you feel comfortable writing a strong letter of recommendation for me for graduate school?"

#### Give them packet containing:

- Photo (at top!)
- List of schools you're applying to and deadlines.
- Statement of purpose
- Summary of every research project.
- Courses/grades in all math/CS/engineering/science
- Extracurricular activities and awards/competitions.



# Keep in mind

It all sounds daunting, but you can do it!!

Be sure that you enjoy research (or plan to be a teacher in college) and really want to undertake a long, in-depth research exploration of one topic!

Good luck!!!!!!



## And a short advertisement

If you're thinking about applying to a Ph.D. program, be sure to check out the University of Washington in Seattle!

http://www.cs.washington.edu/education/grad/prospective.html



## Questions??

#### **Resources:**

Advice by Mike Ernst (UW)

https://homes.cs.washington.edu/~mernst/advice/apply-grad-school.html

Especially document by Mor Harchol-Balter (CMU)

http://www.cs.cmu.edu/~harchol/gradschooltalk.pdf



# Where to apply?

What areas of computing interest me?
What type of degree am I considering? MS? PhD?
Why?

What type of academic climate do I want to study in? Do I have any geographic preferences? Any restrictions?

What are my academic credentials? (GPA, research experience, test scores, communication skills) Who is on the faculty at the school I am applying to? Who

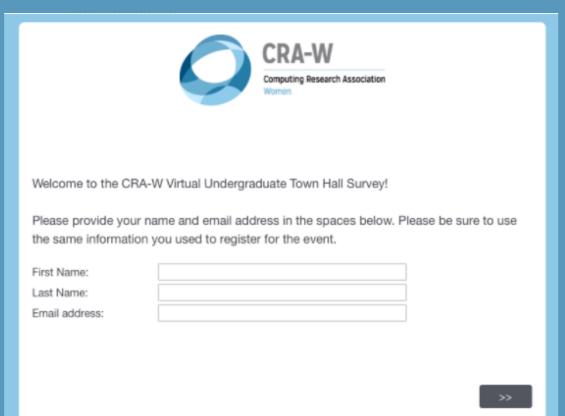
would I like to be my advisor?



# Thank you!

**QUESTIONS?** 

## **Feedback**



Survey URL: http://bit.ly/2nlbT0e



#### Resources

Visit CRA-W.org for more resources for all levels of your career

Join our CRA-W mailing list, CRA-W Updates, by going to bit.ly/1McQCDd

Follow @CRAWomen to find out about upcoming events or programs

Don't forget to take the feedback survey!

PLEASE COMPLETE FEEDBACK SURVEY

Survey URL:

http://bit.ly/2nlbT0e

